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FASTT Math

Fact Fluency Foundations Guide

Laying the Foundation for Math Fact Fluency

By Katherine Garnett, Ed.D.

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Special thanks to Janice Novet, Brooke Barr, and Avia Carmi.

About the Author

The *Fact Fluency Foundations Guide* was conceived and written by Kate Garnett, a leading expert in special education and learning disabilities. Dr. Garnett is currently the chairperson of the Department of Special Education at Hunter College, City University of New York. For more than 25 years, she has been preparing special educators in the graduate program she initiated at Hunter. The heart of Hunter's program—the Learning Lab—is a public service for children from the New York City schools, as well as an intensive preparation for graduate students in learning disabilities. Dr. Garnett's articles on math disabilities have been shared with practitioners through academic journals and are reprinted in resources widely circulated among teachers and parents. Kate speaks and writes with the authority of a master teacher who is conversant both with the relevant research and with the day-to-day realities of students struggling to learn the basics of mathematics.

ISBN 0-545-05971-2

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Note:

Throughout the guide there is reference to the *teacher administation tool*.
If you have *FASTT Math* version 1.2, this tool is **FASTT Math Manager**.
If you have *FASTT Math Enterprise Edition*, this tool is **Scholastic Achievement Manager (SAM)**.

About This Guide

Students achieve math fact fluency when they have both an understanding of the arithmetic operations and automaticity with basic number combinations. Most students construct their concepts of number and learn to count as precursors to the operations of addition, subtraction, multiplication, and division. In their first math experiences, at home and in early schooling, they are developing the thinking that will eventually become the foundations and back-up strategies of counting on, skip counting, counting back, and fact linking—all as part of a natural progression leading to increasingly rapid, and eventually automatic, retrieval.

For a variety of reasons some students lack some, or much, of this conceptual and skill foundation. This *Fact Fluency Foundations Guide* is included with *FASTT Math* as a safety net for such students. The *FASTT Math* software will alert you to those students whose performance on the initial placement quiz indicates a potential gap in foundational number understanding. These students should be further assessed using the *Math Fact Diagnostic Assessment* included in this guide. The diagnostic assessment will classify student needs into those that are predominantly conceptual, those related to insufficient counting skill, or those that are mostly a matter of poor connections between one fact and another. This guide will then direct you to the appropriate set of intervention activities to fill in the gaps and prepare students to use the *FASTT Math* software.

Here's what's included:

Background and Overview

Who are these students and why do they have problems? This section summarizes the issues and offers helpful tips for teaching students with a history of failure in math.

Math Fact Diagnostic Assessment

This one-on-one, interview-based assessment will help you evaluate the gaps in number knowledge, counting, and operations that could be preventing a student from building automaticity and fluency with basic number facts. The results of the interview-based assessment will direct you to the appropriate interventions.

Quantity Concepts Intervention

Very few of your students, if any, may have failed to develop a representation of number quantity. This section offers a structure for addressing this problem.

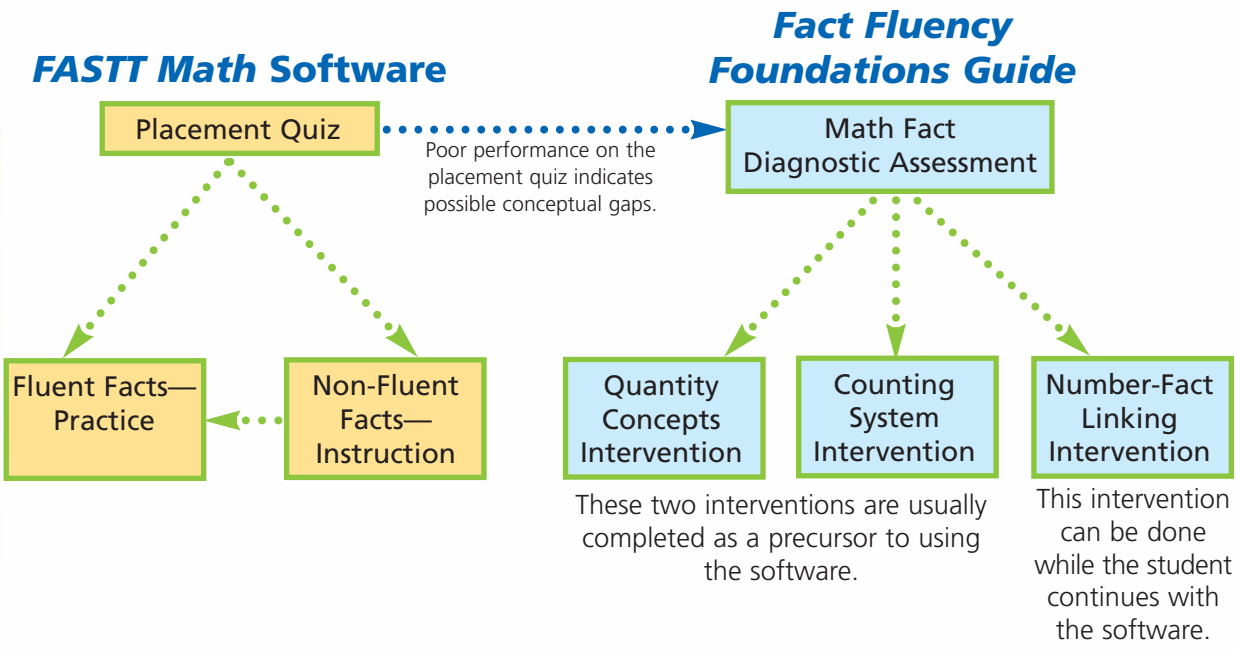
Counting System Intervention

Agility with our number system is critical for math fact fluency. This section includes 15 discrete lessons for building counting skills.

Number-Fact Linking Intervention

This section provides students with the foundation facts and strategies for figuring out new facts from facts they have already learned.

The diagram below shows how the *FASTT Math* software and *Fact Fluency Foundations Guide* work together.



Four steps for successful intervention

1 Have your students complete the Placement Quiz in the *FASTT Math* software and review the results of the low performers.

When students first log in to *FASTT Math*, an initial placement quiz ("Fast Fact Challenge") will identify which facts they know quickly and correctly and which facts they need to learn and automatize. The teacher administration tool for *FASTT Math* will alert you to low-performing students. Review their results. Students who lack speed and accuracy with zeros, ones, and twos may have a fundamental conceptual problem that needs to be addressed prior to or in conjunction with their use of the software.

2 Assess the low-performing students flagged by the software with the *Math Fact Diagnostic Assessment* in this guide.

The interview-based assessment which starts on page 14 will help you pinpoint the underlying issues facing individual students.

3 Use the appropriate intervention(s) highlighted by the *Math Fact Diagnostic Assessment*.

The Diagnostic Assessment will prescribe interventions and activities to address each student's needs. The interventions are described in this guide and the worksheets are available electronically in the teacher administration tool.

4 Re-assess students with the *Math Fact Diagnostic Assessment* and the *FASTT Math* Placement Quiz.

The interventions in this guide represent sequential development and learning. As one intervention is completed, reassess students with the next section of the *Math Fact Diagnostic Assessment*. Once all the necessary interventions are complete, students should retake the *FASTT Math* Placement Quiz to determine an accurate baseline of fact fluency. To trigger the Placement Quiz a second time for a student, go to the Settings tab in the teacher administration tool and unassign the student. Then reassign the student.

Note:

See page iv for information about the teacher administration tool.

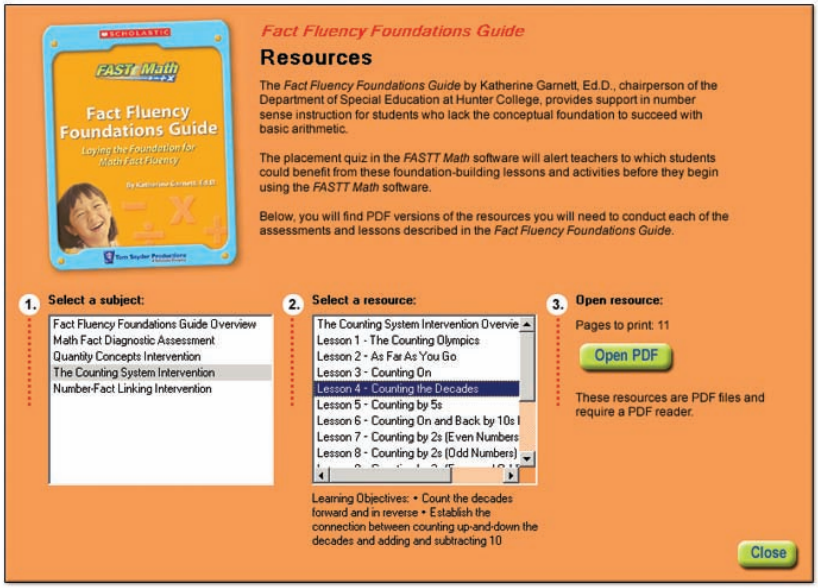
Accessing the lessons and worksheets

A copy of this guide is also available through the teacher administration tool. To make it easy for you to find the worksheets and reproducible materials for each activity, they are organized by lesson in the software. Want to do Lesson 5 of the Counting System Intervention? With just 2 clicks of the mouse, you can have everything you need in one convenient PDF file.

Here's how it works:

1. Launch the teacher administration tool.
2. On the Log In screen, click the *Fact Fluency Foundations Guide* button.

3. A new window will open allowing you to navigate to different sections of the *Fact Fluency Foundations Guide* and interventions. Select a section title from the list on the left.
4. The associated files will appear in the right column. Select one of the files and you will see a short description of the contents.



Worksheets and reproducible materials are easily accessible through the teacher administration tool.

5. To print one of the PDF files (you will need a copy of Adobe Acrobat Reader), click the Open PDF button.
6. The file will open and you can print it.

Notes:

- See page iv for information about the teacher administration tool.
- In *FASTT Math Enterprise Edition*, you can navigate to each section by clicking a bookmark after you open the PDF file of the guide.

Background and Overview

The *Fact Fluency Foundations Guide* offers insights, directions, and lessons to help you form a responsible intervention plan for ensuring that many more of your students make enduring progress in the learning of math. It provides diagnostic portraits, diagnostic assessment, research-based guidelines, and targeted activities. Its focus is foundation mathematical skills; its purpose is the development of robust and flexible number sense in students experiencing significant math learning difficulties.

Three related interventions

The instructional strategies in this guide are targeted interventions for students from late second grade onward who, for whatever reasons, have not solidified a sufficient math foundation and need instruction in one of the following:

Quantity Concepts

Proficiency in this area means that students possess a robust understanding of quantities and quantity relationships. (For example: the "threeness" of 3, the "fiveness" of 5, and the quantity/size/distance relationship of 3 and 5.)

The Counting System

Proficiency in this area means that students possess nimble counting skills for navigating the base-ten number system. (For example: knowing how to count by 10s, by 5s, and how to leapfrog back 5 from 50 on the mental number line.)

Number-Fact Linking

Proficiency in this area means students are capable of readily linking basic number facts into an efficient mental network. (For example: moving in fluid thinking steps to figure 5x6. "*OK, that's 6 fives, and that's up-5 from 5 fives, so, it's 30.*")

Note:

Please note the related adjectives: **robust, nimble**, and **ready**. Students' understandings need to be **robust**, their skills **nimble**, and their connections **readily** activated. If your students have shaky or partial understanding or skills, they may intermittently give the impression that they "know" this stuff, but they may not have developed the mental flexibility, for example, to increase 14 by 10 without counting up one rung at a time. Lack of strong basic understanding and agile skill on the mental number line are evidence of an insufficient foundation. Think about it—to build on it, a foundation must be firm.

How can you use these interventions?

Each of the three interventions is described in greater detail in the pages that follow. You'll find lots of flexibility with the Quantity Concepts and Number-Fact Linking interventions, which include guidelines and materials for you to weave the content into your instructional routine. The Counting System intervention offers a more structured routine over fifteen 20-30 minute lessons that build on one another. You can make use of the activities and lessons provided in these interventions in any of the following ways:

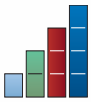
- Replace the grade-level math curriculum for a period of time for students who need intensively focused foundation work
- Supplant part of the grade-level curriculum
- Supplement the ongoing grade-level curriculum
- Serve as a precursor to the *FASTT Math* computer intervention
- Accompany the *FASTT Math* software

Who needs which of these interventions?

The *Math Fact Diagnostic Assessment* section in this guide can help determine which type of intervention—Quantity Concepts, The Counting System, or Number-Fact Linking—is appropriate for which student. The following section, *Math Fact Foundations: Three Kinds of Needs*, provides a brief portrait of different math needs as they relate to these three interventions.

Math Fact Foundations: Three Kinds of Needs

For students to have success in math, they need a foundation that includes spatial, linguistic, and strategic understanding of numbers and their relationships. Students need a mental picture of how much each number represents. They also need mastery of the language of numbers and counting. Finally, they need strategies for computing those numbers and recognizing relationships. Here's a closer look at each of these needs and how the interventions included in this guide address them.

Quantity Concepts Intervention 

Understanding quantities and quantity relationships

This intervention focuses on developing a student's mental representation of quantity (the "threeness" of 3) and the relationships between quantities (increases and decreases). We are born with a sense for number, a sense we hone throughout the early years of life. In large measure, number is a perceived aspect of things in space, so our mental representation of quantity and the increasing and decreasing of quantity is grounded in our processing of spatial information.

For some children, this spatial underpinning for math does not develop well, resulting in their not "getting it" when we try to teach them math in schools. Such students need to go back to physical representations and rebuild this foundation, harnessing their relatively better-developed language skills. Even though they may seem "too old" or "too smart" for this level of instruction, the key to their securing a math foothold is to return to physical number representations (things) and to firmly connect these to verbal representations (words), to written representations (numerals), and to actions on the number line (our system of numbers).

Who needs this intervention?

Students in late second grade and beyond who do not display solid understanding of quantities and quantity relationships are far behind the grade-level curriculum. It is important, and sometimes not easy, to discern the particular needs of these students, who are a relatively small subgroup of the many students experiencing difficulty learning math.

Within an overall school population, students needing this degree of conceptual intervention are uncommon. They generally fall into one of three categories:

- Students with discernible neurological deficits (e.g., traumatic brain injury, Turner's syndrome, cerebral palsy, etc.)
- Students with an overall slow-learning profile
- Students who should be deemed "dyscalculic" (meaning they have a primary math disability, presumed to relate to underlying non-verbal/spatial deficits)

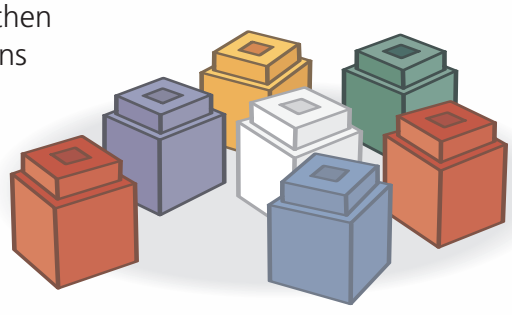
Note:

Students with Dyscalculia

Because children with dyscalculia sometimes do not display obvious characteristics as children with clear neurological deficits do, and because they are not slow-learning overall, the nature of their math learning difficulties is often misunderstood, sometimes grossly misinterpreted, and even overlooked. These students display other areas of strength and talent—commonly in verbal areas. Their areas of strength can divert the unskilled eye from catching the seriousness of their math learning needs. Therefore, it is important to provide in-depth assessment for students who may have a significant mathematics disability and who are, at the same time, linguistically bright or even gifted. Neuropsychological assessment can clarify the nature of their difficulties, a healthy process both for them and for those around them. It can also lead to appropriately focused mathematics intervention—and math accommodations—which can be crucial in their lives.

How does this intervention work?

While all children benefit from work with concrete materials, for students in this subgroup the need is more crucial, as their mental representation of quantities and quantity relationships is an area of deficit. Teachers need to model how to use the manipulatives and verbalize their thought process aloud. They need to offer students many opportunities for repetitive practice until the students can readily show and tell on their own. The work then proceeds, usually slowly, to securing the connections between these actions and words, using math manipulatives and written numerals, written algorithms, and the written number system (counting, number lines, and number grids).



Quantity Concepts and *FASTT Math*

Important!

Students who need this level of intervention are **not ready** for the *FASTT Math* software. Their primary need is to develop understanding. Students demonstrate understanding when they can readily show and explain combining/separating, and comparing/grouping/segregating actions with manipulatives.

The Counting System Intervention

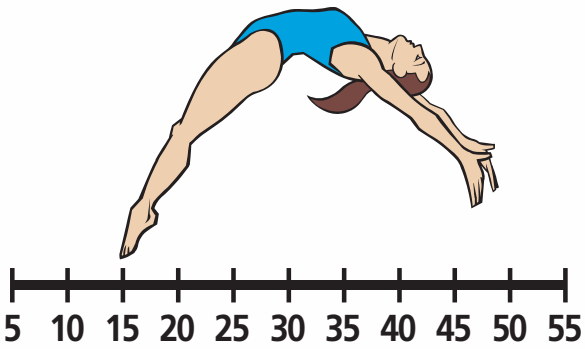
Developing skills for navigating the number system

Well-developed counting skills are critical to a solid foundation for computation. Students need to be facile at navigating a mental number line, progressing forward or backward from any spot, counting by 2s, 5s, and so on. This counting agility provides a base for understanding arithmetic and the operations.

Who needs this intervention?

Many children in late second grade and beyond who perform slowly or inconsistently in basic arithmetic actually lack facility in navigating our system of numbers. We often assume that such students are skilled and nimble counters when, in fact, they are not. Unfortunately, we frequently fail to focus on what these students need most in order to gain a firm grasp on our base-ten number system—we neglect to strengthen and extend their counting skills.

The system inherent in our number system manifests in repeating patterns that are both audible and visible. When children do not "catch" these repeats, it means they don't yet fully grasp this system, resulting in their not developing crucial agility in math thinking. The counting system is akin to a mental jungle gym on which learners need to become adept math gymnasts. They need to swing with ease, up-10 or back-10 from any point on the mental number line, to forward-flip-5 or back-flip-5 from 10, from 20, 45, or 175, to offer just a few agility examples.



Where students break down in the counting system is where their understanding of the system leaves off, so learning to count up (and up and up) is a basic exercise in the Counting System Intervention. The intervention also engages students in counting on, counting back, counting by, counting on by, and counting back by—skills that, when well-exercised, comprise the foundation for what is commonly known as "mental math."

At its outer edges, robust and flexible counting skills blend into mental math—it's about knowing how to "think math" with intuitive number system understanding, which is not just about "remembering" or "being right." Mental math skills engender confidence in young learners—that delighted sense that they *can*. Children feel their mental prowess when they experience their minds thinking their way to a result. This exhilarating boost then feeds into their resilience when new concepts aren't immediately obvious, helping them persevere while figuring out something difficult. Practicing mental math also promotes children's flexible thinking as they travel the many branches of mathematical learning. Thus, in multiple ways, the counting-mental math connection is an important foundation to establish through targeted math intervention.

How does this intervention work?

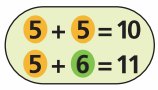
This intervention consists of 15 progressive lessons organized around the theme of a Counting Olympics. The lessons are sequential, building on students' ability to count up and back, to count by, to count out loud, and to count with number lines and number grids. Students follow an up-tempo instructional routine that keeps them moving and improving.

Important!

The Counting System and *FASTT Math*

Some children whose understanding of Quantity Concepts is robust, but who need to work out on the Counting System, can get underway with this intervention as a ramp-up to working with the *FASTT Math* software. For them, a week or so of half-hour sessions on this jungle gym could be sufficient to then combine it with *FASTT Math*, which could well increase motivation and propel progress. For others, a full course of the Counting System workouts—daily half-hour sessions for three weeks or more—is an important prerequisite to the *FASTT Math* software.

Number-Fact Linking Intervention



Strengthening connections between math facts

As students build proficiency and confidence with the Counting System, interconnections among math facts are formed. Number-Fact Linking develops as students increasingly seek and find more efficient ways to answer basic fact problems. Being able to recognize, for instance, that 7+8 is just 1 more than 7+7 not only provides a reliable backup strategy when 7+8 can't be recalled, it reflects a strategy that gets called on when facing bigger numbers, like 127+38.

Who needs this intervention?

Students who need this intervention demonstrate math understanding and counting skill, but are still slow and circuitous when calculating basic number facts. Most teachers view these students as in need of "memorizing" their facts and, indeed, direct retrieval from memory may be an appropriate goal. But, it is equally important to develop rich networks of linked facts and nimble activation of these as shortcuts. Thus, this Number-Fact Linking Intervention can be a useful ramp-up—or an early accompaniment—to the *FASTT Math* software.

How does this intervention work?

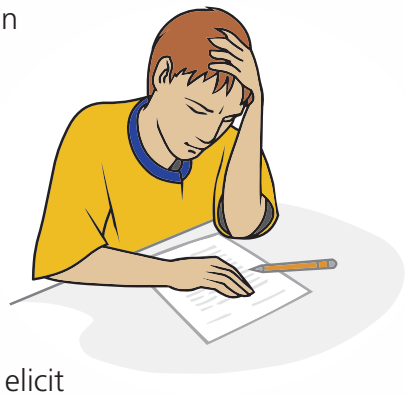
The Number-Fact Linking Intervention makes the interconnections among facts explicit, highlighting specific strategies for thinking-from-what-you-know in order to develop a resilient network of facts. The stronger the linkages of this mental network, the more secure the "backup" system beneath arithmetic calculating and reasoning. A robust backup system is important as a bridge to our original number understandings, which sink below the surface of awareness as math facts and processes are automatized.

Number-Fact Linking and FASTT Math

The Number-Fact Linking Intervention is particularly well suited to work in concert with *FASTT Math*. In fact, they make a particularly effective duo, serving both to strengthen direct retrieval of math facts and to reinforce math fact network connections.

Tips on Working with Students in Need of Intervention

Students who need any of the three math foundation interventions, Quantity Concepts, the Counting System, or Number-Fact Linking, are not performing well in comparison with their peers (and, you can be sure, they make those comparisons!). As they lose their mathematical footing in late second grade and beyond, not only do they lag behind classmates, they also learn lessons that are often hard to unlearn, like:



My brain doesn't work…
Effort doesn't pay…
I can't…
I don't like this…I don't want this…I don't need this

At the start of any intervention, as you focus on what kids really need in the way of "basics," it is common to elicit signs and symptoms of underlying *I can't* or *I won't* scenarios. Students beginning intervention can be hyper-reactive to peer putdowns, teacher irony—or even teacher praise—as these can be viewed as the continuation of previous frustration, embarrassment, or humiliation. Students may raise barriers to learning, such as:

- Digging in their heels (resisting to tread where they've already fallen)
- Derailing instruction
- Denigrating activities as "stupid" or "boring"
- Side-tracking
- Investing minimal attention
- Giving up

Such self-protective moves require skilled handling so as not to further undercut an already eroded sense of self-efficacy. There are several keys to providing a safe and encouraging arena for interventions:

Progress charting

Real progress, even in small increments, is a potent motivator. Being able to experience distinct steps forward is key, which is why progress charting is so effective—it offers clear, visual evidence that a student's efforts are effective. Successfully navigating a number line, completing number grids and worksheets, and progressively filling in the fact grid in the *FASTT Math* software all provide students with measurable signs of progress that feed self-confidence and further effort.

Reinforcement and feedback

Regular descriptive acknowledgment from others—in this case, teachers—provides the most profound sort of support by focusing encouragement. Such descriptive support requires knowing that what the student has done is actually a step forward, is heading in the right direction, or is what you want more of. And discerning this is not nearly as easy as it sounds; it requires clear-sightedness about what you're aiming to teach step-by-step, not just overall. Clearly describing what the learner has just done successfully is beneficial in at least two significant ways. It demonstrates to the student that you see (a fundamental reinforcement for us humans) and it highlights what was worth seeing (helping the student attend to what was most important). Again, however small a step that action may have been, noting it invites the student to congratulate him/herself—*yes, I did do that…and you noticed!*

Skillful descriptive acknowledgment is one of our most powerful teaching tools, pointing the way and fueling the learner's courage to take the next step. Unfortunately, too many of us fall into using "empty-praise" phrases such as *Good! Good! Excellent!* in our attempts at motivating students. Too often such well-meant praise teaches nothing, mis-focuses the learner, and actually backfires as students begin to not believe it. Be specific and timely. *You paused and listened to yourself—good strategy!* Being explicit helps students recognize what they have learned and use it later.

No putdowns

A safe social arena is key to taking the risks of learning (e.g., venturing into unknown territory). Peer putdowns—blatant or subtle, verbal or just a look or snigger—make risk-taking many times more difficult. Putdowns are a major hindrance to classroom learning. If allowed to infect small-group intervention, they can halt progress. Fumigating for putdowns can be as easy as being clear, firm, and alert to every instance. Or it can require more, such as teaching children how to praise and encourage one another, inserting competing behavior where putdowns used to be. By whatever creative means, putdowns need to be eliminated.

Math Fact Diagnostic Assessment

Who needs this diagnostic interview?

There are a couple of ways to determine if a student needs this diagnostic interview:

1. You've observed a student and have concerns that he or she has yet to establish a basic number sense or counting strategies. You can use this assessment to explore the student's needs.
2. You've logged into the teacher administration tool and received an "alert" in the Message Center indicating a student's low performance on the initial placement quiz and possible insufficient understanding of numbers and number relationships. The alert will direct you to further examine that student's skill and knowledge.

Note:

See page iv for information about the teacher administration tool.

Purpose

The purpose of this assessment is to uncover students' lack of skill and understanding in order to determine their needs for intervention. When students do not master basic number facts in a timely fashion, we tend to serve up more fact practice, neglecting to look underneath at more fundamental understandings and skills that may need shoring up. With this assessment, you can probe this underlying foundation for the learning of math facts in addition, subtraction, multiplication, and division.

How is this assessment organized?

This diagnostic math interview is divided into three parts corresponding to the three interventions in this guide:

Quantity Concepts

Understanding quantity and quantity relationships

Students needing intervention at this level are likely to be rare in comparison with those needing to strengthen their counting system foundation. They require highly concrete, language-intensified instruction, tailored to their particular needs. This section of the assessment looks at student ability to relate number and quantity.

The Counting System

Developing skills for navigating the number system

Students requiring this intervention need to strengthen their underlying counting skills and connect these to moves on an extended number line. These skills are the stepping-stones for mental math. Students needing to strengthen their counting and number line skills are more common than most teachers realize. This section of the assessment examines student nimbleness with counting, forward and backward, by different increments and from different starting points.

Number-Fact Linking

Strengthening connections between math facts

Students in need of this intervention have not elaborated their connections between math facts. They need to increase their direct retrieval of some facts while strengthening the connections among related facts. These many connections then form mental networks that facilitate shortcut strategies, help monitor accuracy, and that act as backup recall when needed. This section of the assessment is only for students you believe are ready for multiplication and division.

Administering this assessment

Gather Materials

- A digital timer (stopwatch)
- A photocopy of the *Math Fact Diagnostic Assessment* protocol (pages 18–25)
- A handful of loose paperclips
- 3 paperclip strings (each with 10 paperclips strung together)
- 0–50 Number Line with only the decades marked (page 26)
- Reading Numerals sheet (page 27)

Organize space and time

Familiarize yourself with the assessment format, questions, activities, and recording sheets. Organize the materials so that they will be handy when you need them. Find a place where you can work with a student quietly and without interruption for up to 20 minutes.

Start at the beginning

Assessment of each student should begin with the Quantity Concepts Assessment Item #1 and proceed in order. The student who experiences no difficulty with Quantity Concepts should speed along and complete the section in under two minutes, proceeding immediately to the second section, the Counting System.

Administer individually

This math diagnostic interview should be administered by a teacher or qualified aide individually to one student at a time.

Administer aloud

In no way is this assessment a paper and pencil "test." It is a probing diagnostic interview, aimed at determining an individual student's math needs. Present it orally, accepting oral responses from the students. In addition to oral math being much like real-life math situations, this verbal interaction is also a significant window into what the student really knows and can do.

Do not let it drag

Take care not to let this assessment go on too long. When you observe that a student can only complete a task via tedious counting or laborious strategies, you need not prolong that item. Cut a task short when it becomes clear that the student doesn't know it, doesn't understand it, or can only accomplish it laboriously.

For "Count by" items (10–13 in the Counting System Assessment) the directions specify, *"Stop at ___, if accurate."* This means that if the student is counting accurately and swiftly, go to the prescribed limit (which won't take much time at all). If the student is slow or error-prone, stop earlier than that limit, as soon as competent performance seems to be breaking down.

Note breakdown points

It is important to note what the student can and cannot do and to identify where skills break down (e.g., he can count like lightning to 100, but then leaps to 200). It is also important to note laborious strategies the student uses (e.g., fingers-tapping-mouth to count each addend).

Note other performance characteristics

Sometimes it is important to note accompanying aspects of the student's performance—qualitative indicators related to self-regulation, language, and even emotional response. The emotional signals comprise a legitimate focus of intervention unto themselves (e.g., we can, and often should, teach children to be more effortful, more willing, and less reactive). As you observe the student's interactions with the tasks, note anything that seems important and might relate to how you target your follow-up intervention activities.

Here are some performance indicators to watch for:

Evidence of problems related to self-regulation

- Lack of awareness of errors
- Impulsive errors, with or without self-correction
- Difficulty concentrating and staying focused
- Problems returning to focus
- Rigid adherence to one inefficient procedure
- Difficulty shifting from one item to another

Evidence of language-based difficulties

- Underuse of self-talk to guide performance
- Confusion about the teacher's language use
- Misuse or underuse of math terminology
- Confused explanations
- Word and name retrieval difficulties

Evidence of emotional responsiveness and frustration

- Self-anger
- Denigrating task
- Sidetracking
- Anger at others
- Somatizing (tiredness, stomachache, headache)
- Sadness/hopelessness
- Shut down

It is important to note what the student can and cannot do and to identify where skills break down.

Math Fact Diagnostic Assessment

Teacher Name: _____
Student Name: _____
Student Age: _____
Class/Grade: _____
Interviewer: _____
Date: _____

Quantity Concepts **Assessment Items** **Instructions: Say the words in *italics***	Notes on Performance
1. Say: ***Show on your fingers*:** a. ***2 plus 3***… Then, ***How much is that?*** b. ***4 plus 5***… Then, ***How much is that?*** c. ***6 plus 4***… Then, ***How much is that?***	❑ Proficient ❑ Not Proficient **Proficiency means doing the following for items a, b, & c:** • showing total number of fingers without counting them individually • answering "how much" without counting the fingers again
2. Say: ***Is 6 closer to 12 or to 10?***	❑ Proficient ❑ Not Proficient **Proficiency means answering "10" with assurance after a brief thinking pause.**
3. Put 9 paperclips on the table. a. Point to the 9 paperclips and say: ***Count these***. b. Say: ***Show me 7 paperclips***. c. Say: ***With the paperclips, show me 3 plus 4***.	❑ Proficient ❑ Not Proficient **Proficiency means:** a. Counts accurately and easily. b. Does not recount, but shows all but two. c. Splits the 7 paperclips easily into 3 and 4, without counting each paperclip in each group.
4. Say: ***Is 9 closer to 3 or to 11?***	❑ Proficient ❑ Not Proficient **Proficiency means answering "11" with assurance after a brief thinking pause.**
5. Prepare 3 strings of 10 paperclips each. a. Provide 1 string of paperclips. Say: ***Here is a string of paperclips. How many paperclips are there?*** b. Provide 2 more 10-strings. Say: ***How many paperclips are there now?*** c. Take back one 10-string and say: ***How many paperclips are there now?***	❑ Proficient ❑ Not Proficient **Proficiency means:** a. Counts paperclips accurately and easily. b. Answers "30" without counting individual paperclips. c. Answers "20" without counting individual paperclips.
6. Say: ***Is 4 closer to 9 or to 12?***	❑ Proficient ❑ Not Proficient **Proficiency means answering "9" with assurance after a brief thinking pause.**

7. Lay out a number line with only 10, 20, 30, 40, and 50 showing (see reproducible on page 26).

a. Say: *Can you show me where we could put the number 13 on this number line?*

b. Say: *Where could we put 15?*

c. Say: *Where could we put 25?*

d. Say: *Where could we put 36?*

❑ Proficient ❑ Not Proficient

Proficiency means doing the following for items a, b, c, & d:

- Performs accurately and easily, without counting up from one.

Proficient on items 1–6, then continue Assessment

If the student has shown proficiency with items 1–6, it is likely that this student does not need the Quantity Concepts Intervention. If the student only experiences difficulty with the number line tasks (item 7), then proceed to the Counting System Assessment Items (pages 20–23).

Not proficient on items 1–6, provide Quantity Concepts Intervention

For students who demonstrate difficulty with the concepts sampled above, provide instruction offered in the Quantity Concepts Intervention. See pages 28–37 of this guide. Since the Quantity Concepts portion of the assessment only briefly samples performance, you may want to further probe aspects of these students' math knowledge (e.g., reading and writing numerals and number sentences, math vocabulary, counting skills) to be able to include these in your Quantity Concepts Intervention.

You may also choose to further investigate these students' skills by continuing to administer the Counting System Assessment. If you do so, take care not to press the student to demonstrate skills not yet developed. Once you can discern a breakdown point or confusion, either proceed to the next type of item or conclude the assessment.

Math Fact Diagnostic Assessment

Teacher Name: _____
Student Name: _____
Student Age: _____
Class/Grade: _____
Interviewer: _____
Date: _____

The Counting System Assessment Items	Notes on Performance
Numerals **1.** Show student one line of numerals at a time on the Reading Numerals reproducible (see page 27). For each line, say: *Read these numerals*: a. 5, 8, 12, 17 b. 29, 68, 85 c. 103, 152, 210, 574 d. 1,021; 1,604	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
Counting **2.** Say: ***Count for me starting at 1.*** Have student start with 1 and stop at 115, if accurate. Stop earlier if inaccurate.	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
For all counting-on (items 3–5), have student count for only a short stretch of numbers. **3.** Say: ***Count, starting with 7*** *(to about 15)* **4.** Say: ***Count, starting with 18*** *(to about 25)* **5.** Say: ***Count, starting with 57*** *(to about 65)*	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
6. Say: ***Count, starting with 94*** *(to about 105)* **7.** Say: ***Count, starting with 189*** *(to about 201)* **8.** Say: ***Count, starting with 539*** *(to about 545)* **9.** Say: ***Count, starting with 996*** *(to about 1005)*	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.

10. Say: ***Count by 2s.*** Have student start with 2 and stop at 40, if accurate. Stop earlier if inaccurate.	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
11. Say: ***Count by 10s.*** Have student start with 10 and stop at 110, if accurate. Stop earlier if inaccurate.	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
12. Say: ***Count by 5s.*** Have student start with 5 and stop at 75, if accurate. Stop earlier if inaccurate.	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
13. Say: ***Count by 25s.*** Have student start with 25 and stop at 325, if accurate. Stop earlier if inaccurate.	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.
14. Say: ***Count backward from 15*** *(go all the way to 1)* **15.** Say: ***Count backward from 31*** *(to about 18)* **16.** Say: ***Count backward from 63*** *(to about 48)*	❑ Proficient ❑ Not Proficient Note errors, confusion, or breakdowns in performance.

Mental Math Say: *What is*: **17.** *6-1* **18.** *12-1* **19.** *70-1* **20.** *150-1*	❑ Accurate & rapid ❑ Accurate with counting ❑ Inaccurate/confused *Note specific errors of -1 performance*
Say: *What is*: **21.** *6+2* **22.** *2+14* **23.** *26-2* **24.** *35+2*	❑ Accurate & rapid ❑ Accurate with counting ❑ Inaccurate/confused *Note specific errors of +/-2 performance*
Say: *What is*: **25.** *10+7* **26.** *25+10* **27.** *33-10* **28.** *10+41*	❑ Accurate & rapid ❑ Accurate with counting ❑ Inaccurate/confused *Note specific errors of +/-10 performance*
Say: *What is*: **29.** *5+20* **30.** *45+5* **31.** *35-5* **32.** *85+5*	❑ Accurate & rapid ❑ Accurate with counting ❑ Inaccurate/confused *Note specific errors of +/-5 performance*
Say: *What is*: **33.** *25+25* **34.** *125-25* **35.** *25+50* **36.** *75+25*	❑ Accurate & rapid ❑ Accurate with counting ❑ Inaccurate/confused *Note specific errors of +/-25 performance*

Say: *What is*:	❑ Accurate & rapid
37. *11+11*	❑ Accurate with counting
38. *20+20*	❑ Inaccurate/confused
39. *12+12*	*Note specific errors of +11, +20, and +12 performance*

For students working on addition and subtraction

Proficient (few difficulties) on items 1–39, continue with *FASTT Math* software
Note: There are no specific Number-Fact Linking Assessment items for addition and subtraction. Therefore, if the students are proficient overall with counting (items 1–16) and mental math (items 17–39), they may not need additional intervention and could proceed with the *FASTT Math* software.

Proficient (few difficulties) on items 1–16 but labored on items 17–39, go to Number-Fact Linking Intervention
If students are still slow and labored with addition and subtraction facts, then they are candidates for the Number-Fact Linking Intervention (beginning on page 108 of this guide) without need for further assessment.

Not Proficient (many difficulties) with items 1–39, provide the Counting System Intervention
If students stumble particularly with items 1–16, they likely need the entire instructional sequence offered in the Counting System Intervention. Even students with select difficulties among the counting system items can strengthen their skills by proceeding through all the Counting System Intervention lessons. See pages 38–107 of this guide.

For students ready for multiplication and division

Proficient (few difficulties) on items 1–39, proceed to Number-Fact Linking Assessment on the following page

Proficient (few difficulties) on items 1–16 but labored on items 17–39, go to Number-Fact Linking Intervention
If students are still slow and labored with addition and subtraction facts, then they are candidates for the addition and subtraction sections of Number-Fact Linking Intervention (beginning on page 108 of this guide) without need for further assessment. You should consider assigning these students to the addition component of the *FASTT Math* software and having them progress through subtraction before beginning multiplication.

Not Proficient (many difficulties) with items 1–39, go to the Counting System Intervention
If students stumble particularly with items 1–23, they likely need the entire instructional sequence offered in the Counting System Intervention. Even students with select difficulties among the counting system items can strengthen their skills by proceeding through all the Counting System Intervention lessons. See pages 38–107 of this guide. You should consider assigning these students to the addition component of the *FASTT Math* software and having them progress through subtraction before beginning multiplication.

Math Fact Diagnostic Assessment

Teacher Name: _____
Student Name: _____
Student Age: _____
Class/Grade: _____
Interviewer: _____
Date: _____

Number-Fact Linking **Assessment Items—MULTIPLICATION ONLY** **Instructions: Time each item, allowing up to 3 seconds.**	Notes on Performance **Tell your student that he or she will have a short amount of time (3 seconds) to answer each question. If the answer does not come quickly (within 3 seconds), tell your student to "wipe that one away" and get ready for the next one.**
Say: *What is*: **1.** *2 x 9* **2.** *2 x 7* **3.** *8 x 2*	❑ Accurate & within 3 seconds ❑ Longer than 3 seconds ❑ Inaccurate *Note specific errors of x2 performance*
Say: *What is*: **4.** *8 x 5* **5.** *5 x 6* **6.** *7 x 5*	❑ Accurate & within 3 seconds ❑ Longer than 3 seconds ❑ Inaccurate *Note specific errors of x5 performance*
Say: *What is*: **7.** *6 x 9* **8.** *9 x 4* **9.** *9 x 8*	❑ Accurate & within 3 seconds ❑ Longer than 3 seconds ❑ Inaccurate *Note specific errors of x9 performance*
Say: *What is*: **10.** *8 x 8* **11.** *4 x 4* **12.** *7 x 7*	❑ Accurate & within 3 seconds ❑ Longer than 3 seconds ❑ Inaccurate *Note specific errors of perfect squares performance*

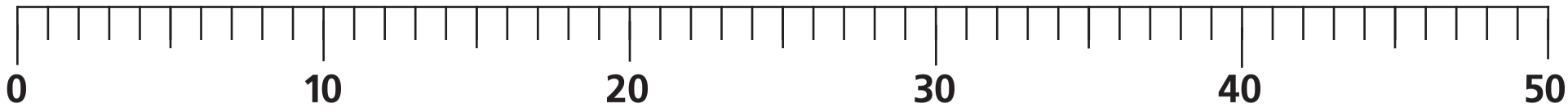
Proficient (at least 9 out of 12 items within 3 seconds), proceed with *FASTT Math* software

Not Proficient (slow or incorrect on most or all of the items above), provide Number-Fact Linking Intervention

Provide the instruction offered in the Number-Fact Linking Intervention for multiplication (followed by division). See pages 108–119 of this guide. This intervention can be used concurrently with the *FASTT Math* software.

0–50 Number Line

For use with Quantity Concepts Assessment item 7.



Reading Numerals

For use with the Counting System Assessment item 1.
Fold or cover parts of the sheet to display each line for students to read.

a. **5** **8** **12** **17**

b. **29** **68** **85**

c. **103** **152** **210** **574**

d. **1,021** **1,604**

Quantity Concepts Intervention

Understanding quantity & quantity relationships

Quantity concept development can be slow going for students who need to develop this level of conceptual underpinning at older ages. It can require an attentive focus on seeing and feeling the "fourness" of 4 and the "eightness" of 8, for example, and on calibrating the quantity and the distance between them. These students may also need to learn to verbalize what they are doing as they compare quantities, order them from smaller to larger, and match them to a running number line and 100-grid, repeatedly over time. The teacher's modeling of actions and related words, followed by the student's saying-and-doing, serve to direct and focus the learner's attention. Pairing actions and concrete materials together with clear verbal describing is an important duet for developing their robust mental representation of quantity and the relationships among different quantities.

Using manipulatives

For students needing this intervention, it is important to employ one type of structured concrete material (such as Unifix cubes or Stern rods) to establish all the basic concepts until students demonstrate and verbalize with that material easily. Additionally, because color is a powerful conceptual support when used strategically, it is important to maintain consistency of color usage (the three-cube stack always being maroon, the five-cube stack always being red, for example). When using a math manipulative such as Unifix that can be separated into individual cubes, it is also important to keep the cubes attached, even taped together, to establish a stable mental representation.

Once concepts can be readily demonstrated using one material across the range of actions (combining, separating, comparing, ordering, grouping, regrouping), then you can, and should, branch out, covering the same territory with other materials—for example, Dienes blocks, paperclips, chips, etc.

Note:

Students with math concept deficits

It is important to draw a distinction here between the use and value of tangible math manipulatives for all students and for those few with math concept deficits. There is tremendous benefit to using math manipulatives for *all* students as they develop math concepts at each stage of the K–8 math curriculum. This general value of clarifying with concrete math materials is different, however, from using such materials with students who have fundamental difficulty establishing basic math concepts. The difference is in the details—in the attention to language, consistency, repetition, and persistent focus. The intervention provided here is aimed at this latter subgroup of students. The guidelines offered for working with these students include, of necessity, close attention to particular instructional details.

Adapting an instructional routine

It is difficult to predict which methods of instruction will spark understanding for the few students who will require this intervention. In this first lesson, you are looking for what "clicks"—the mix of language, verbalizations, and manipulatives that help these students develop a sense of quantity. Providing repeated demonstrations and verbalizations as well as having the student show and tell are keys to securing this foundation sense. You will then need to adapt what you are teaching, how you approach it and for how long to meet the individual child's needs.

With that in mind, review the Sample Routine and Progression of Concepts on the following pages. The intervention outlines an instructional sequence, a basic routine you might follow, and suggested materials. As you work with your students, you will need to use your discretion to adapt the format to the needs of each particular child.

This intervention is not laid out in "lessons," but rather in a progression of areas that need to be practiced until they are understood. Securing these foundation concepts is not a one-time or a once-in-a-while demonstration. There need to be repeated instructional routines that include "*Show me*," "*Now tell me*," "*Now hold it and make an image in your mind*," "*Now show me on the number line*," "*Now write it in numbers*." The teacher's modeling and remodeling and the student's frequent re-enacting of these routines eventually become internalized spatial-verbal math understanding.

Teacher preparation

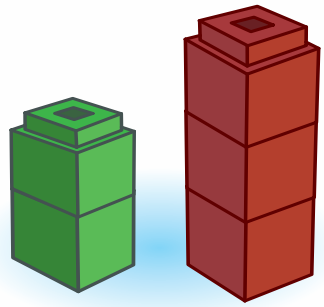
Much of this intervention involves using a consistent set of manipulatives that match a growing number line. It's best to gather and/or create these materials ahead of time. For your convenience, we have included templates (available as PDFs) for making your own number lines, matching manipulatives, and numeral cards.

Gather materials

- 1–10 structured math manipulatives that are color consistent (10 of each stack)
  Note: The particular colors are not important, but choosing a particular color to correspond to a particular number is crucial.

  Here's a sample color scheme:

1 - light blue	6 - orange
2 - green	7 - yellow
3 - maroon	8 - brown
4 - dark blue	9 - white
5 - red	10 - black



Tip:

Color Consistent Math Manipulatives

It is important in teaching all of the concepts in this intervention that you use color-consistent math manipulatives that map to the number line. For example, all "2 stacks" should be the same color (green, for instance), all "3 stacks" should be another color (say, maroon), etc. Using math manipulatives that are color-number consistent is critical; while establishing a stable mental representation, color-number mixtures and variations can create confusion. To create manipulatives that are color consistent, you can snap (and tape) together Unifix cubes, or you can print out the PDF template on heavy paper stock, color, and cut out. In either case, the manipulatives should "map" to the number line, with one unit of manipulative fitting into a unit space on the number line.

Print and assemble:

- 1–10 Number Line
- 1–20 Number Line
- 1–100 Number Line
  (A blank number line template is included as a PDF file for you to print out and create your own number lines, if needed.)
- Numeral Cards
  (Print and cut out the Numeral Cards PDFs, or create with index cards.)
- 1–100 Number Cards
  (Print and cut up the 1–100 Number Cards PDF and use with the Numeral Cards to build number sentences.)

Accessing the PDF files

The PDF files for the Quantity Concepts Intervention and all needed materials can be accessed through the teacher administration tool (see page 4 for instructions).

Note:

See page iv for information about the teacher administration tool.

Follow the Sample Routine

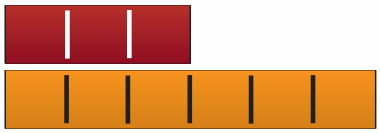
Plan for brief frequent sessions—ideally, five times per week for intensive 15-minute sessions or three times per week for 20–30 minutes. This particular intervention is likely to be carried out one-to-one or in a very small group, so pacing, repeating, and emphasis should be adjusted to the individual learner. Here is a general sequence of activities for each session: Compare, Order, Map, and Connect.

To give you a feel for how this routine might play out, one elaborated example of the routine is provided on the next page, along with a suggestion of how much time to spend on each part. Using these same instructional moves—comparing, ordering, mapping, and connecting—you will progress through the foundation concepts of this intervention at the pace appropriate to the student.

Sample Routine: Quantities 1–10

Step 1: Compare structured math manipulatives (3 minutes)

a. The teacher models how to compare the math manipulatives. First, line up the color-consistent stacks against one another and feel how much longer one is than another. Then, talk about the differences, modeling how to describe the quantity relationships for the student. "*The red-5-stack is two longer than the maroon-3-stack,*" and "*The orange-6-stack is three longer than the maroon-3-stack.*" The purpose is to get a multi-sensory feel for each quantity—the fiveness of 5, the sixness of 6, and the threeness of 3, for example—to feel, see, and verbalize the relationship.



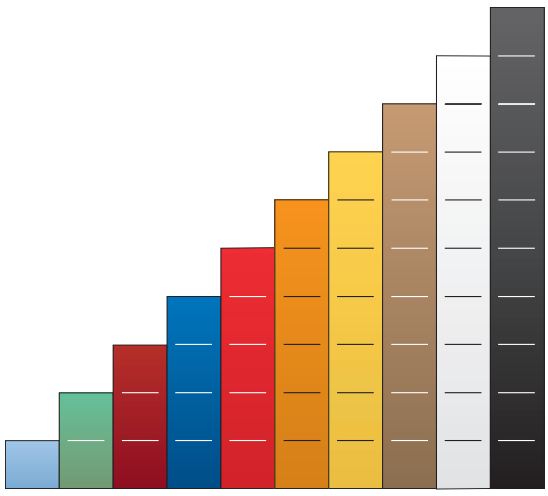
b. Student makes his or her own comparisons and follows the same process of feeling and talking about the differences.

c. Ask student to show and describe relationships between other number stacks.

d. Focus on the vocabulary: More/less, most/least, larger/smaller number, farther/closer.

Step 2: Order structured math manipulatives (3 minutes)

a. The teacher models how to put the color-consistent stacks in ascending order, from the shortest stacks to the longest. The stacks should resemble a staircase. Tell the student, "*Let's now put the stacks in order from the shortest to the longest.*"



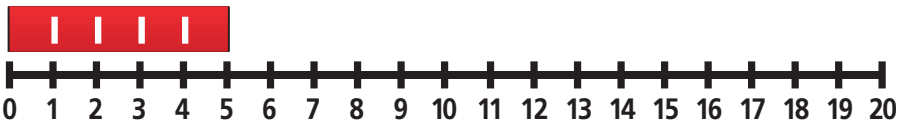
b. Talk about the differences, modeling how to describe the "one-moreness" of each stack. *"The green stack is one more than the light blue stack,"* and *"The maroon stack is one more than the green stack."*

c. Student puts the color-consistent stacks in ascending order and describes the differences.

d. Ask student to show and describe "one-more" relationships between other number stacks.

Step 3: Map to the number line (5 minutes)

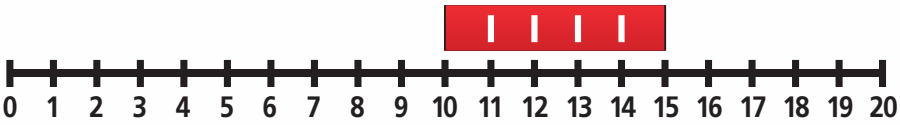
a. The teacher models lining up the manipulatives against the number line. Go in order, from the shortest stacks to the longest, and describe the relationship. *"When I put this stack on the number line, see—it takes up five spaces on the number line."*



b. Verbalize what you are mapping, talking about the increases and decreases as you illustrate with different stacks. This exercise is drawing very concrete and explicit connections among the physical representation of quantity, the number line, and the verbal description.

c. The student lines up the manipulatives against the number line and describes the relationships.

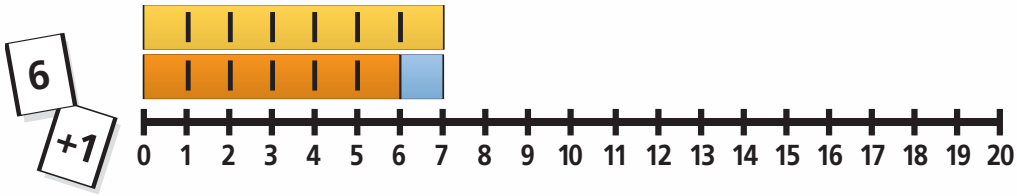
d. Use different starting numbers, continuing to demonstrate and verbalize the relationships.



Step 4: Connect with written numerals and number sentences (5 minutes)

a. Start by matching numeral cards to the corresponding manipulative. "*Here's a 6 card, and here's an orange-6-stack.*" Show how it fits onto the number line.

b. Use the + and - numeral cards to build number sentences. "*6+1. That's an orange-6-stack plus a light blue-1-stack. Together, it's the same as a yellow-7-stack.*" Show how it fits on the number line. You want the student to make the connection between physical quantities (math manipulatives) and the corresponding written notation.



For each concept, remember to:

- Scaffold for the student, modeling actions and verbalizing
- Have the student emulate both your actions and verbalization
- Decrease scaffolding, as the student demonstrates understanding
- Have the student demonstrate and verbalize multiple examples of each concept responding to your repeated queries:

Show me. Now tell me.
Hold it and make an image in your mind.
Show me on the number line. Tell it to me in words.
Now close your eyes and make a number line image in your mind.
Write that in numbers (or in a number sentence).
Now, show me this (written problem) with your cubes.
Now show it to me on the number line.

Continue work on each concept until the student seems to have a stable internal representation of the quantity relationships, how they map onto the number line, and how they translate into the numerals and symbols of number sentences.

Progression of Concepts

The concepts listed on pages 35–37 should be taught in sequence as each concept builds upon the previous one. Each concept should be practiced until the student shows a secure, consistent grasp of it—which may take more than one session.

Concept 1: Quantities 1–10

Description:	Routine:	Materials:
Students work with stacks of varying lengths, 1–10. This session is outlined in the sample routine on pages 32–34 of this guide.	1. Compare quantities 1–10 by look and feel. 2. Order quantities 1–10 less to more by look and feel. 3. Map quantities 1–10 to the 1–10 number line from different starting numbers. 4. Connect manipulatives to written numerals and number sentences.	• color-consistent math manipulatives • 1–10 Number Line • Numeral Cards • 1–10 Number Cards (use with Numeral Cards to build number sentences)

Concept 2: Quantities 1–20

Description:	Routine:	Materials:
Students work with stacks of varying lengths, 1–20 (you will need to combine stacks to create lengths 11–20).	1. Compare quantities 1–20 by look and feel. 2. Order quantities 1–20 less to more by look and feel. 3. Map quantities 1–20 to the 1–20 number line from different starting numbers. 4. Connect manipulatives to written numerals and number sentences.	• color-consistent math manipulatives • 1–20 Number Line • Numeral Cards • 1–20 Number Cards (to build number sentences)

Concept 3: Plus and Minus 10 at the Decades

Description:	Routine:	Materials:
Student uses ten 10-stacks on the 1–100 number line to build a feel for the "tenness" of 10. Student practices adding and subtracting 10 at the decades.	1. Compare quantities 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 by combining 10-stack manipulatives. 2. Order manipulatives to build a feel for 10 more and 10 less (i.e., 30, 40, 50). 3. Map +10 math manipulatives to the 1–100 number line at the decades (i.e., 10, 20, 30, 40, 50, 60, 70, 80, 90, 100). 4. Map -10 math manipulatives to the 1–100 number line at the decades. (i.e., 10, 20, 30, 40, 50, 60, 70, 80, 90, 100). 5. Connect manipulatives to written numerals and number sentences.	• color-consistent math manipulatives • 1–100 Number Line • Numeral Cards • 1–100 Number Cards (to build number sentences)

Concept 4: Plus and Minus (within 5-scale)

Description:	Routine:	Materials:
Student uses twenty 5-stacks on 1–100 number line to build a feel for multiples of 5. Student practices adding and subtracting 5 at points on the 5-scale.	1. Compare multiples of 5, 10, 15, 20 and so on by combining 5-stack manipulatives. 2. Order manipulatives to build a feel for 5 more and 5 less. 3. Map +5 manipulatives to the 1–100 number line on the 5-scale (i.e., 5, 10, 15, 20, 25, 30, 35, 40, etc.) 4. Map -5 manipulatives to the 1–100 number line on the 5-scale (i.e., 5, 10, 15, 20, 25, 30, 35, 40, etc.). 5. Connect manipulatives to written numerals and number sentences.	• 20 color-consistent 5-stack math manipulatives • 1–100 Number Line • Numeral Cards • 1–100 Number Cards (to use with +/-5 and +/-10 Numeral Cards)

Concept 5: Plus and Minus 1 and 2

Description:	Routine:	Materials:
Student uses number stacks to increase and decrease by 1 and 2 on the 1–100 number line at different starting points. Begin the exercise with the 1–10 stacks and progress to using larger, combination stacks (11 and greater).	1. Compare quantities of 1 more, 2 more, 1 less, and 2 less by combining 1-stack and 2-stack manipulatives with other stacks. 2. Order the quantities you created by look and feel. 3. Map +1 and +2 manipulatives to the 1–100 number line at different starting numbers. 4. Map -1 and -2 manipulatives to the 1–100 number line at different starting numbers. 5. Connect manipulatives to written numerals and number sentences.	• color-consistent math manipulatives • 1–100 Number Line • Numeral Cards • 1–100 Number Cards (to use with Numeral Cards)

Note:

Concepts 6 and 7

Students will Map and Connect, but not Compare and Order in developing Concepts 6 and 7. By this point, the student has graduated to knowing that the physical quantity of, say, 8 is represented by the 8 on the number line. Students should not progress to Concepts 6 and 7 until they understand the relationships between physical quantities and their corresponding number stack representations. Concepts 6 and 7 assume that students understand that starting at a specific point on the number line means that amount, without their having to lay out the corresponding stack on the number line.

Concept 6: Plus and Minus 10 from Different Starting Points

Description:	Routine:	Materials:
Student uses one 10-stack and slides it up and down the number line from any starting point to determine the +10 quantity. For example, the student might begin the 10-stack at 8 on the number line, extending to 18 (+10) and then to 28 (+10 again), or begin at 53 and end up at 43 (-10).	1. Map +10 manipulatives to the 1–100 number line at different starting numbers. 2. Map -10 manipulatives to the 1–100 number line at different starting numbers. 3. Connect manipulatives to written numerals and number sentences.	• color-consistent math manipulatives • 1–100 Number Line • Numeral Cards • 1–100 Number Cards

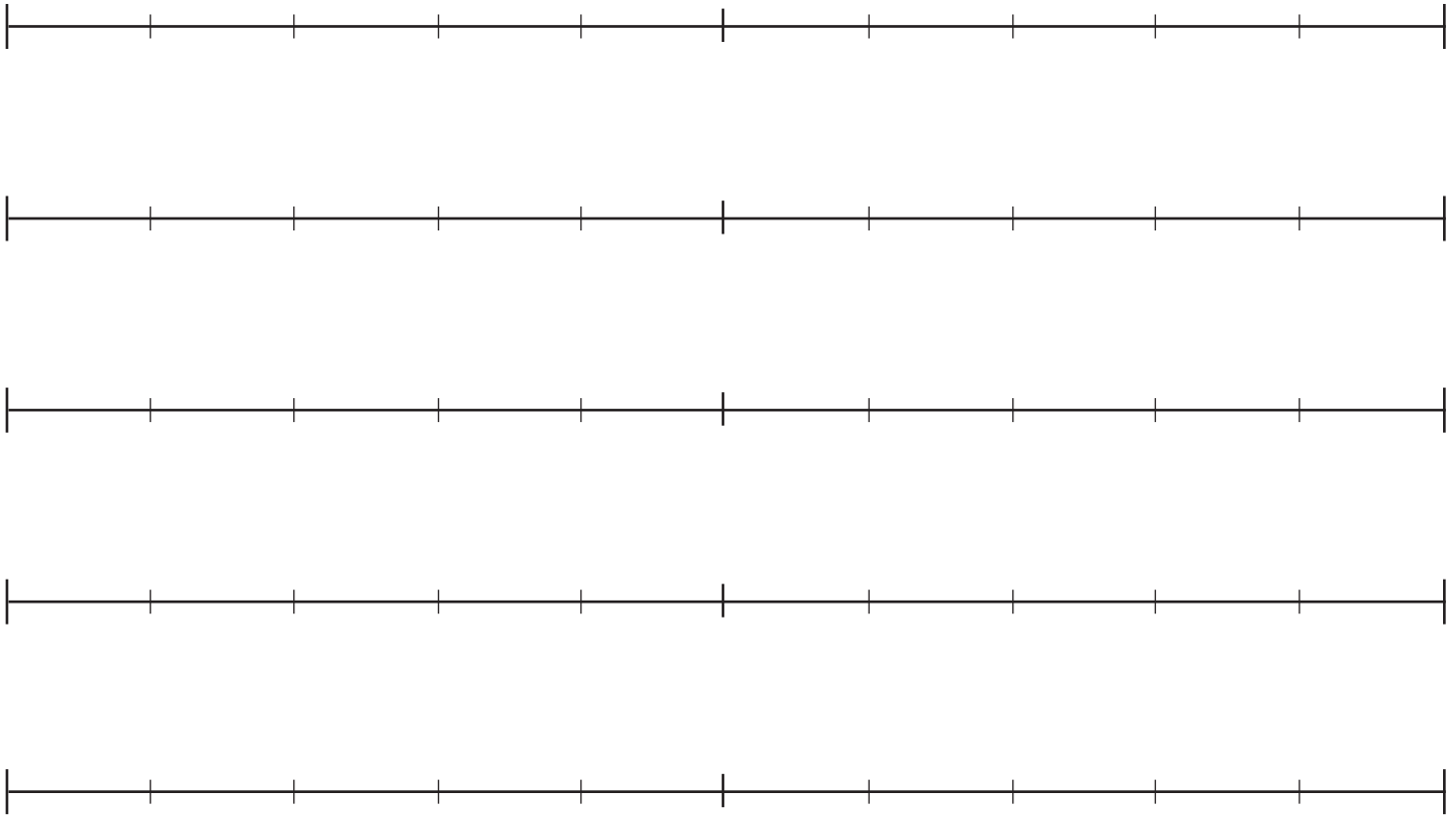
Concept 7: Plus and Minus 1–10 from Different Starting Points

Description:	Routine:	Materials:
Student uses stacks of varying lengths, 1–10, and slides them up and down the number line from any starting point to determine the resulting quantity. For example, the 6-stack might be placed at 16 on the number line and extend to 22, thereby adding 6. Or the stack might begin at 78 and subtract 6, ending up at 72 on the number line.	1. Map +1 through +10 manipulatives to the 1–100 number line at different starting numbers. 2. Map -1 through -10 manipulatives to the 1–100 number line at different starting numbers. 3. Connect manipulatives to written numerals and number sentences.	• color-consistent math manipulatives • 1–100 Number Line • Numeral Cards • 1–100 Number Cards

Blank Number Line

Name: _____

Date: _____

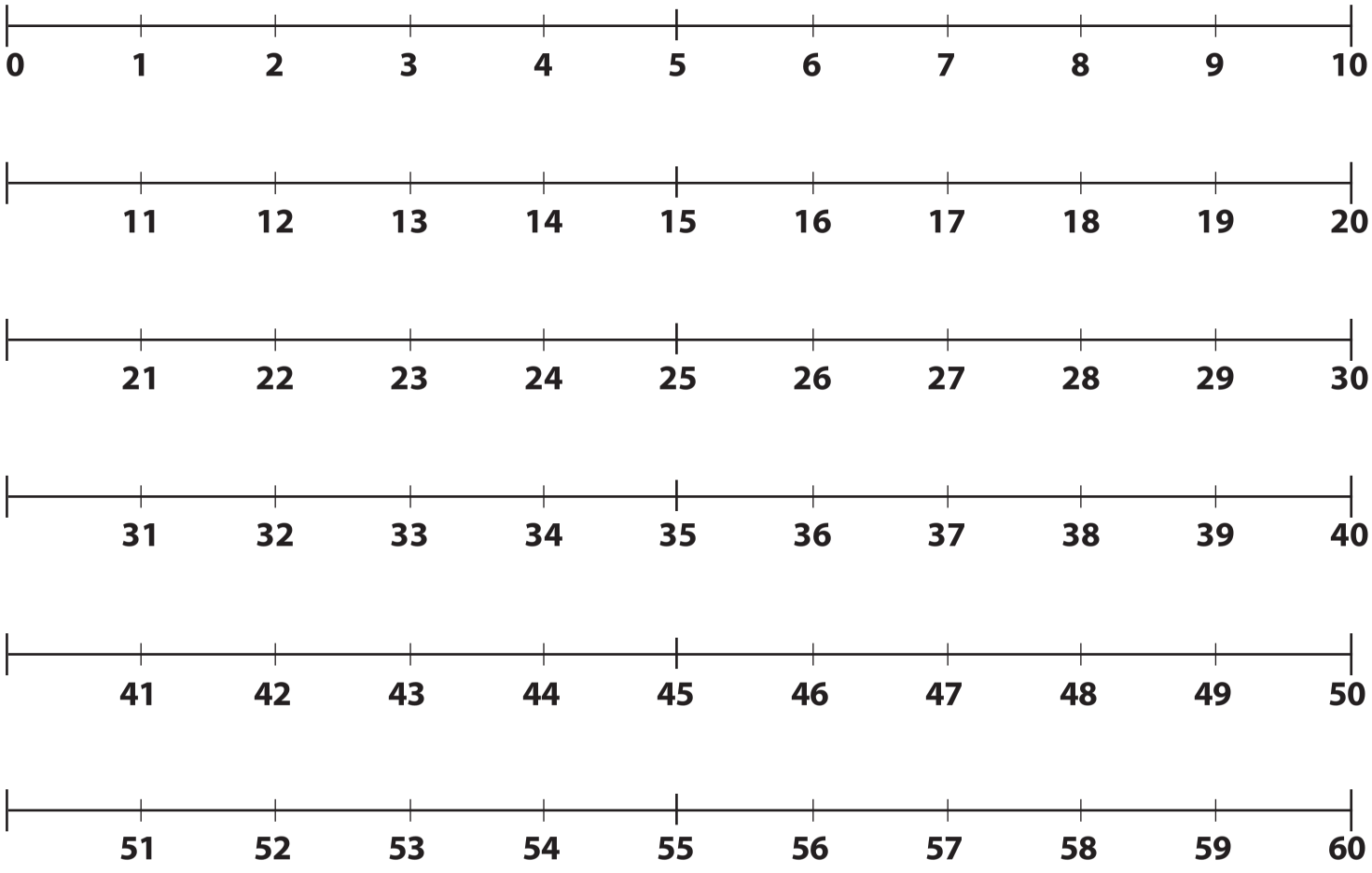


FAST Math +−÷×

Number Line: 0-60

Name: _____

Date: _____



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Number Line: 61-120

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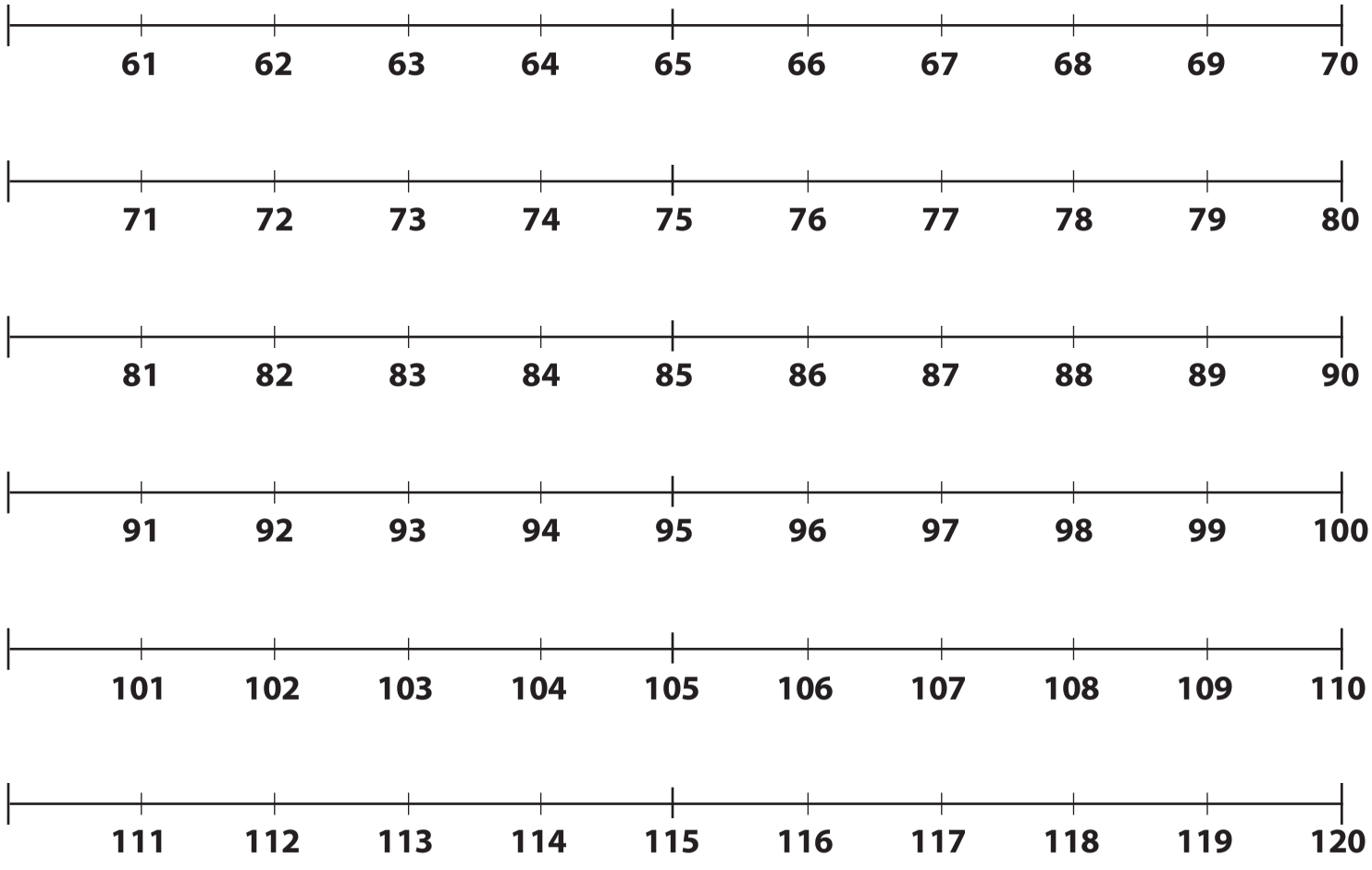
Date: _____



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Number Line: 121-180

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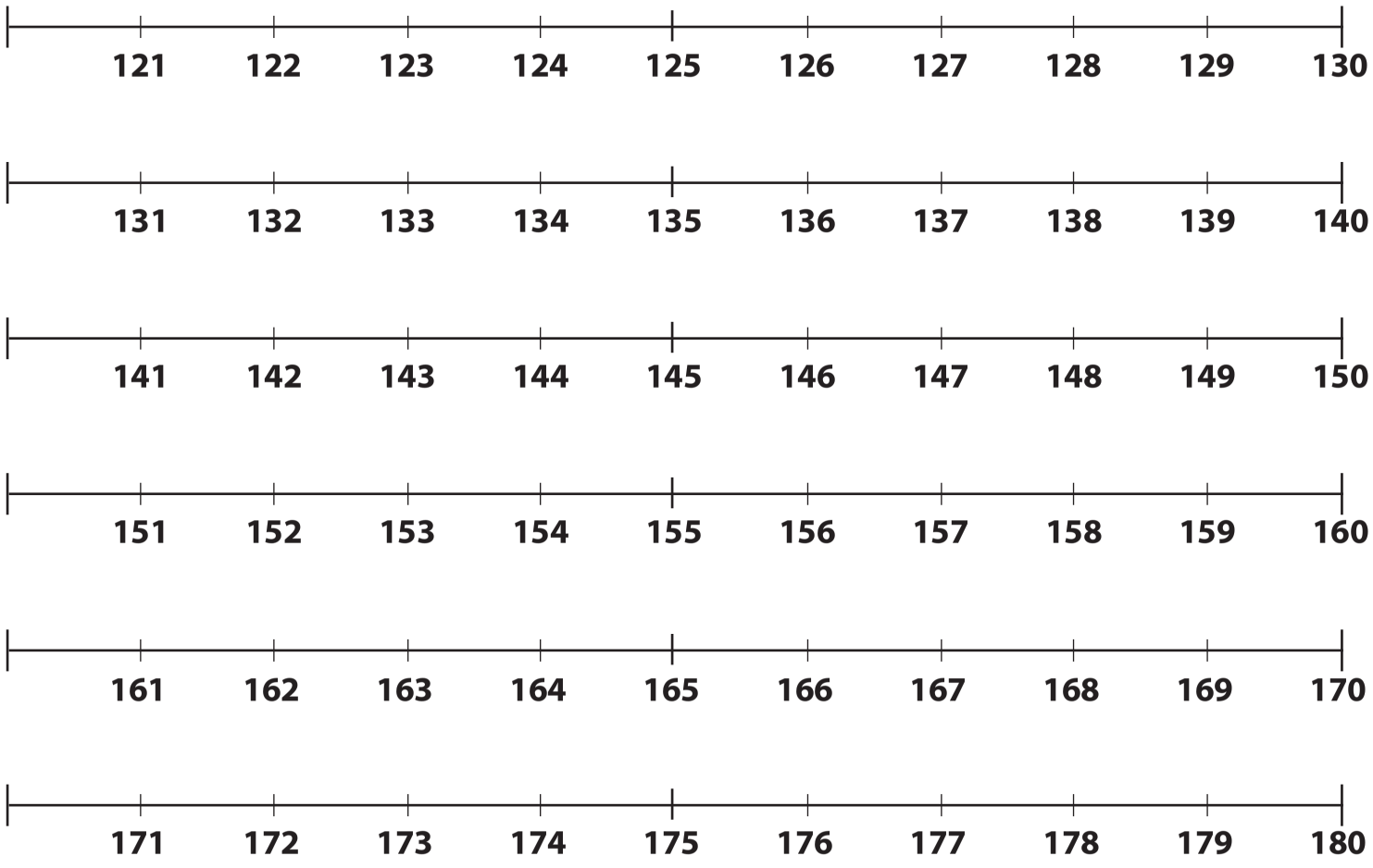
Date: _____



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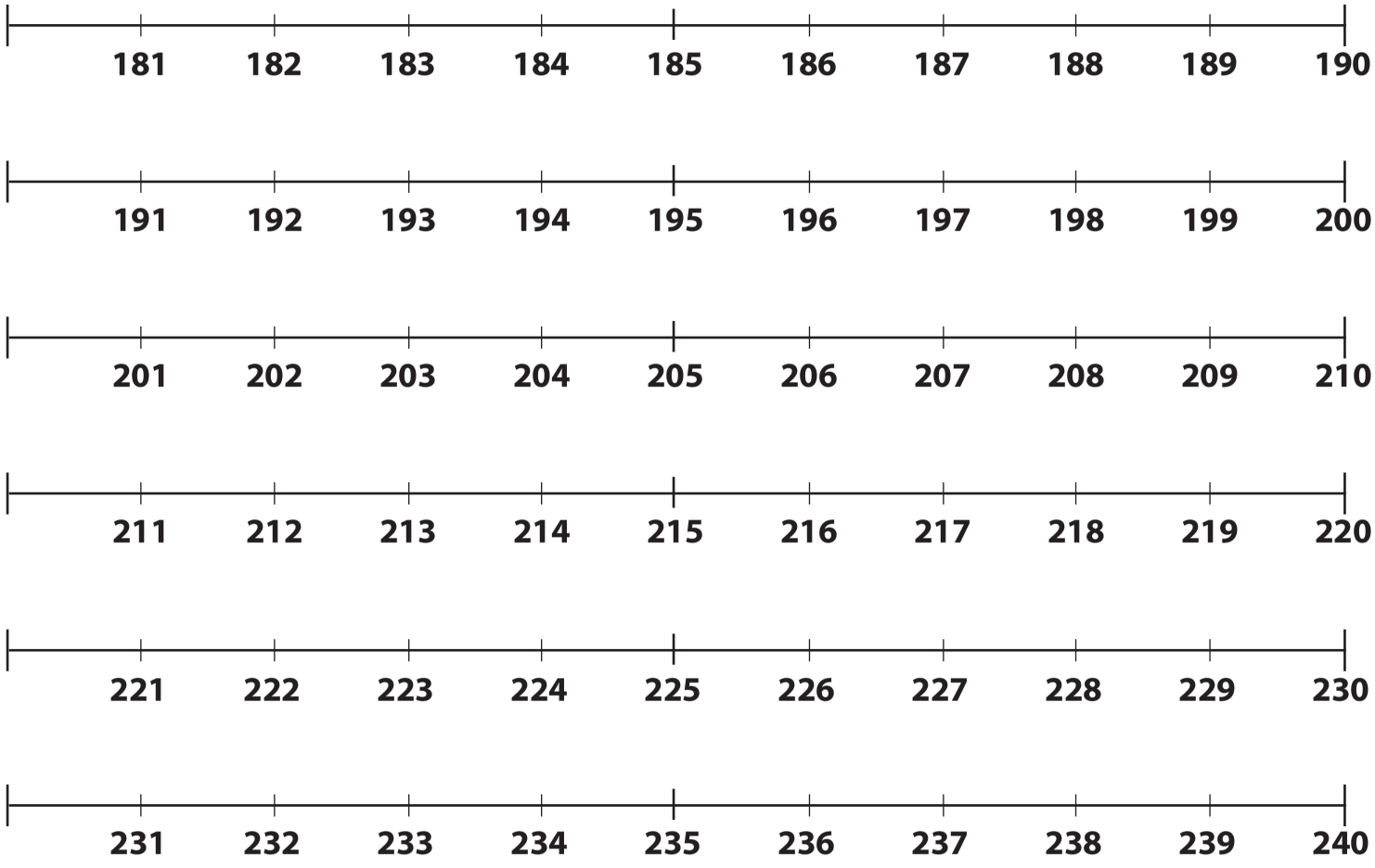
Number Line: 181-240

Name: _____

Date: _____



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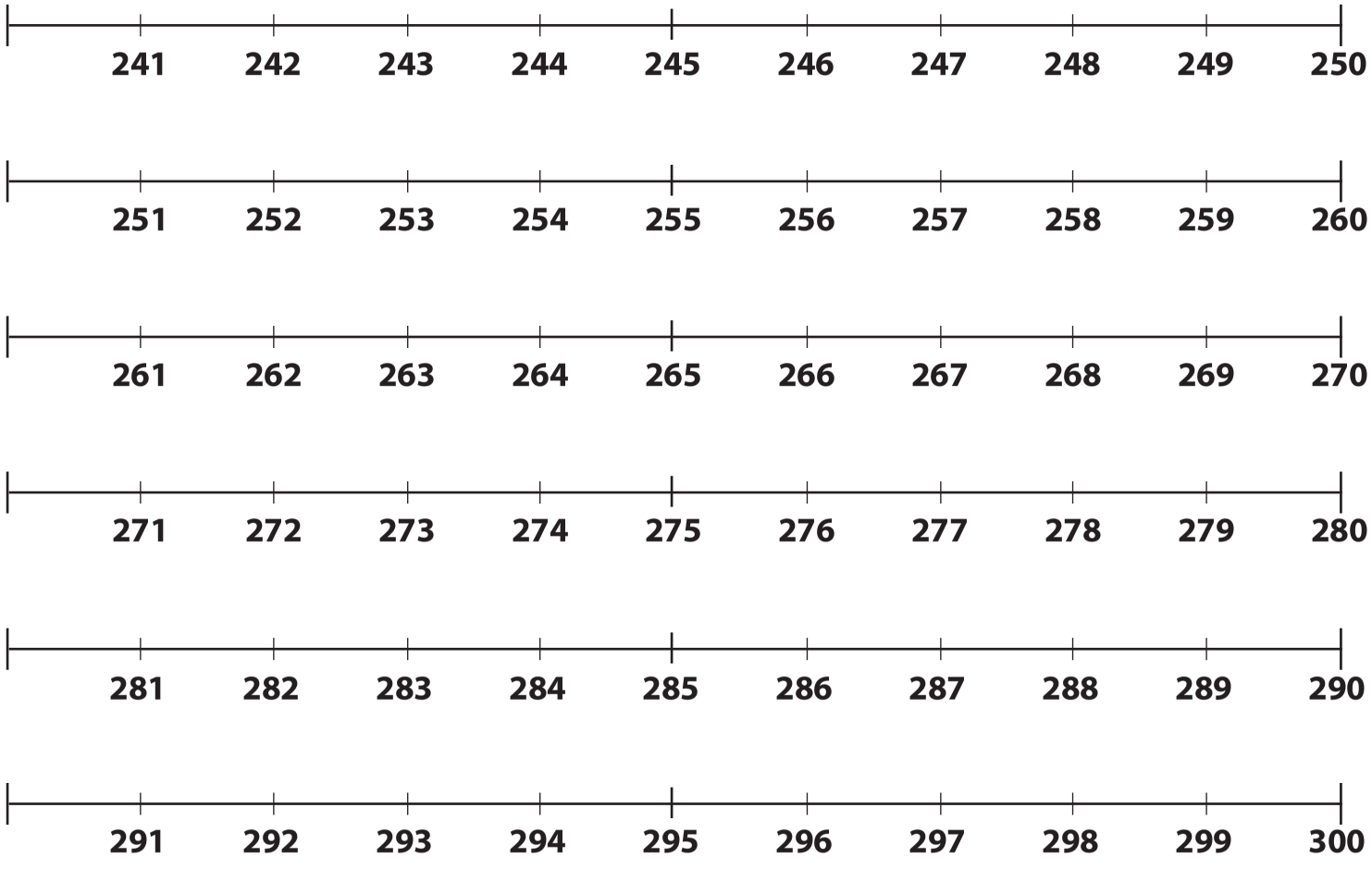
Number Line: 241-300

Name: _____

Date: _____



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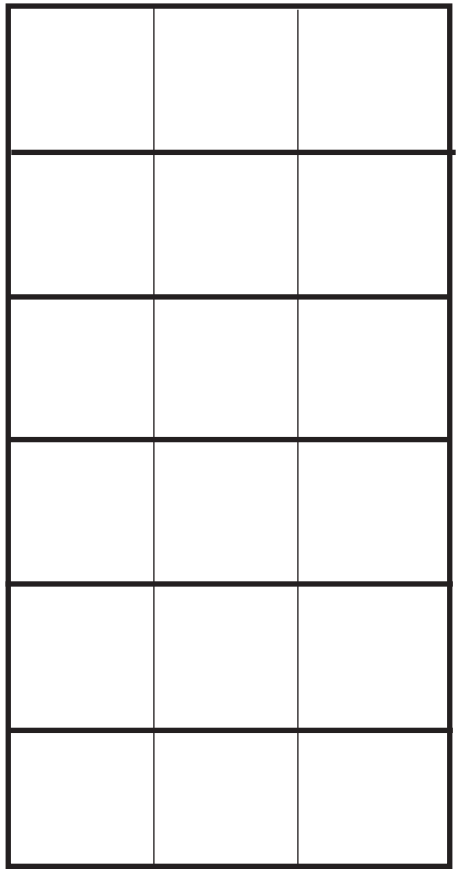
Name: _____

Date: _____

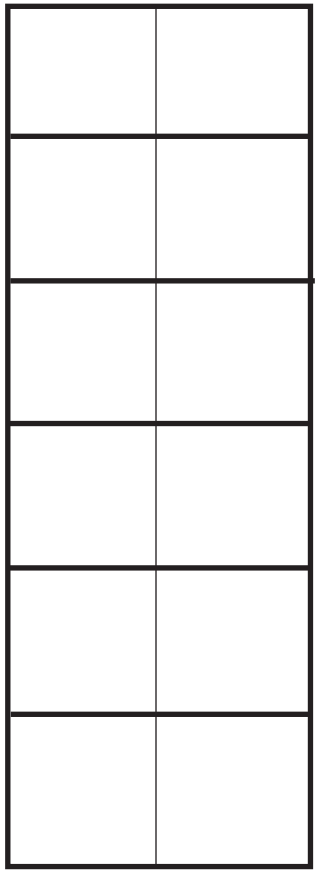
3-Stacks

Choose a different color for the 3-stacks.
Color and cut them out.



2-Stacks

Choose a different color for the 2-stacks.
Color and cut them out.



1-Stacks

Choose a different color for the 1-stacks.
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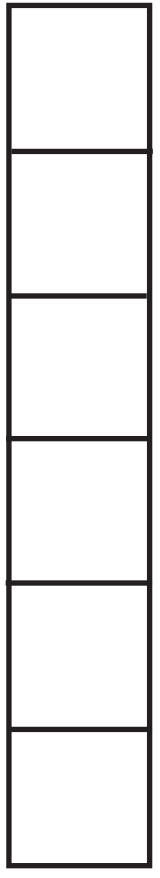


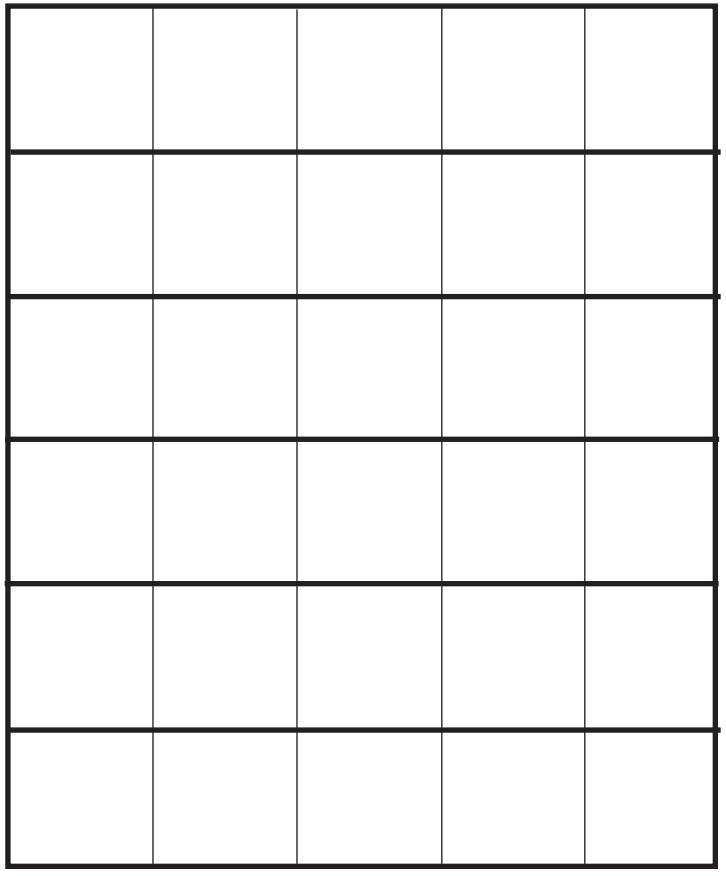
FAST Math +−÷x

Name: _____

Date: _____

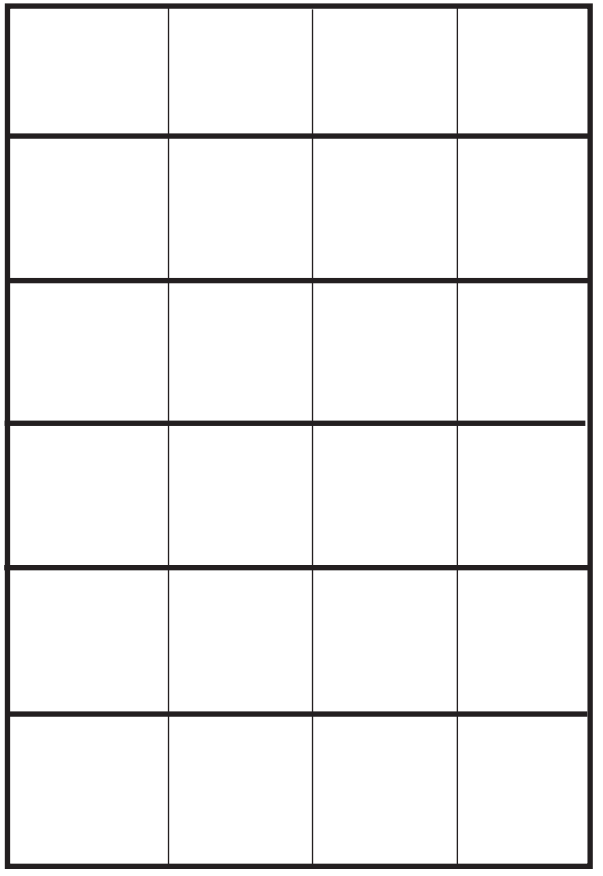
5-Stacks

Choose a different color for the 5-stacks.
Color and cut them out.



4-Stacks

Choose a different color for the 4-stacks.
Color and cut them out.



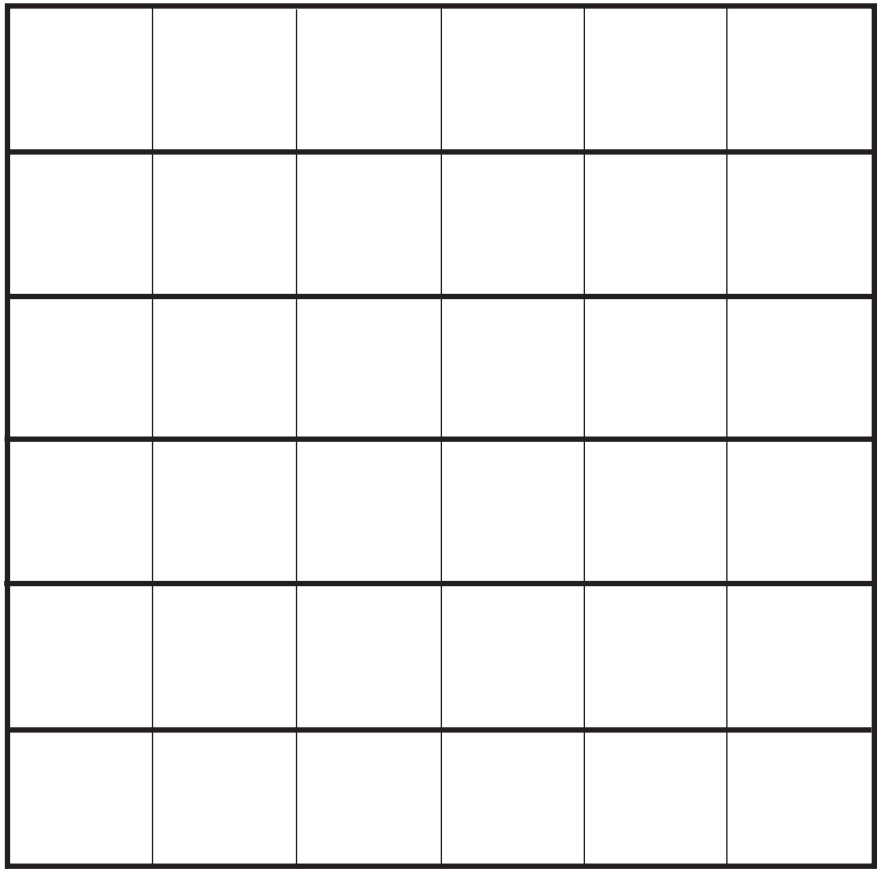
FAST Math +−÷x

6-Stacks

Name: _____

Date: _____

Choose a different color for the 6-stacks. Color and cut them out.



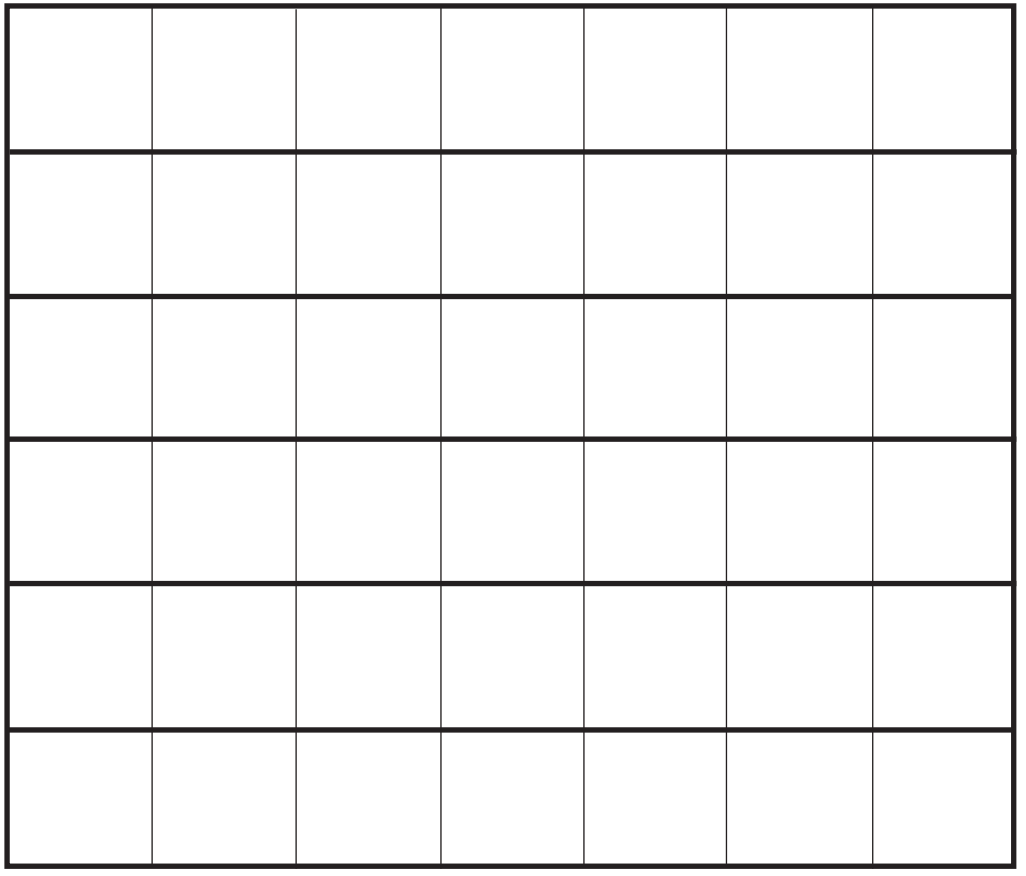
FAST Math +−÷×

7-Stacks

Name: _____

Date: _____

Choose a different color for the 7-stacks. Color and cut them out.



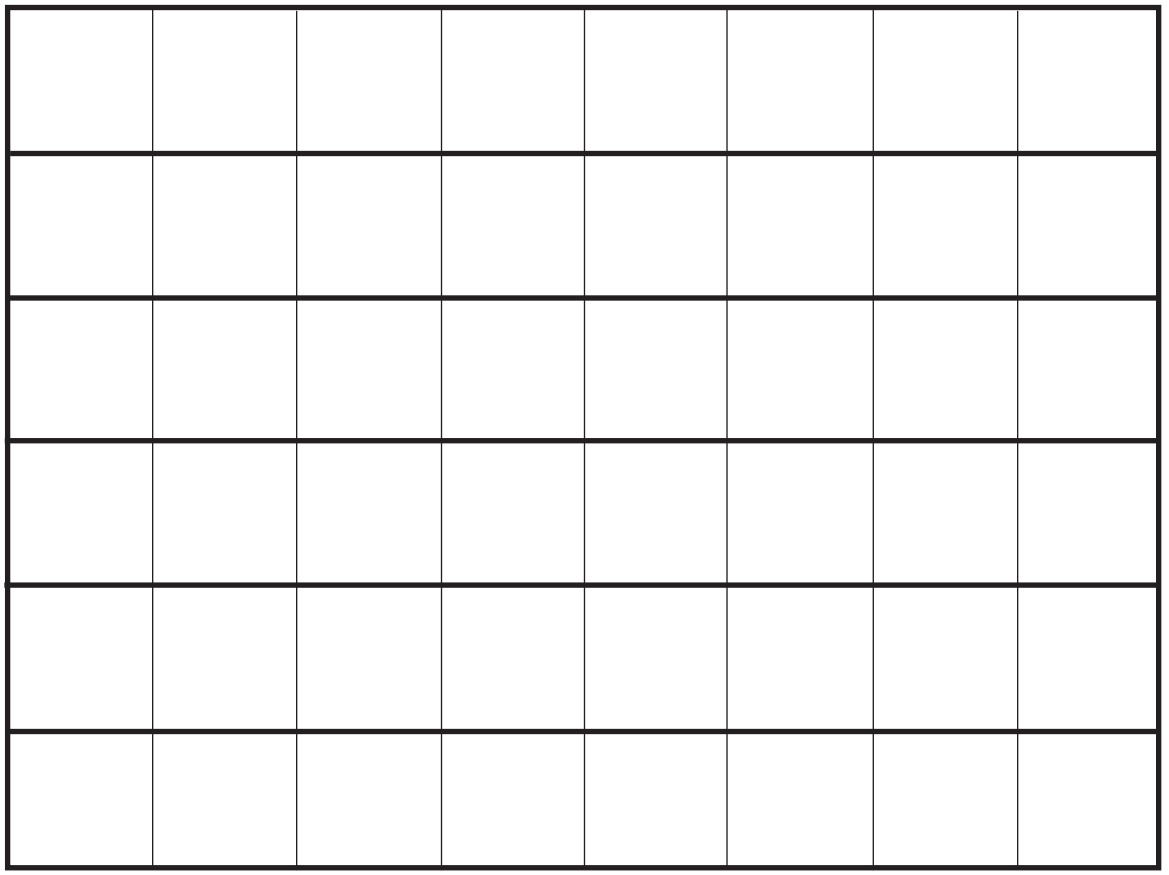
FAST Math +−÷x

8-Stacks

Name: _____

Date: _____

Choose a different color for the 8-stacks. Color and cut them out.



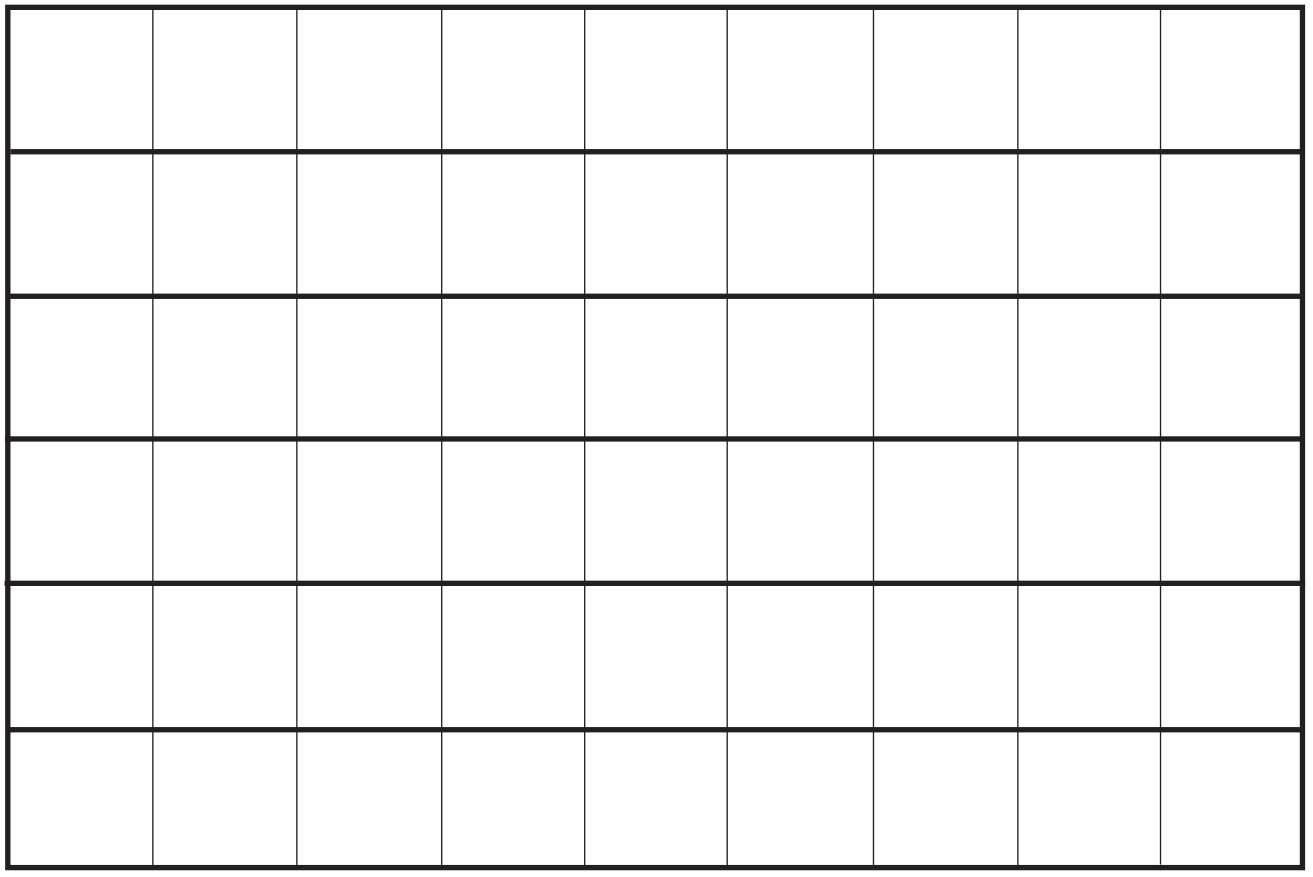
FAST Math +−÷×

9-Stacks

Name: _____

Date: _____

Choose a different color for the 9-stacks. Color and cut them out.



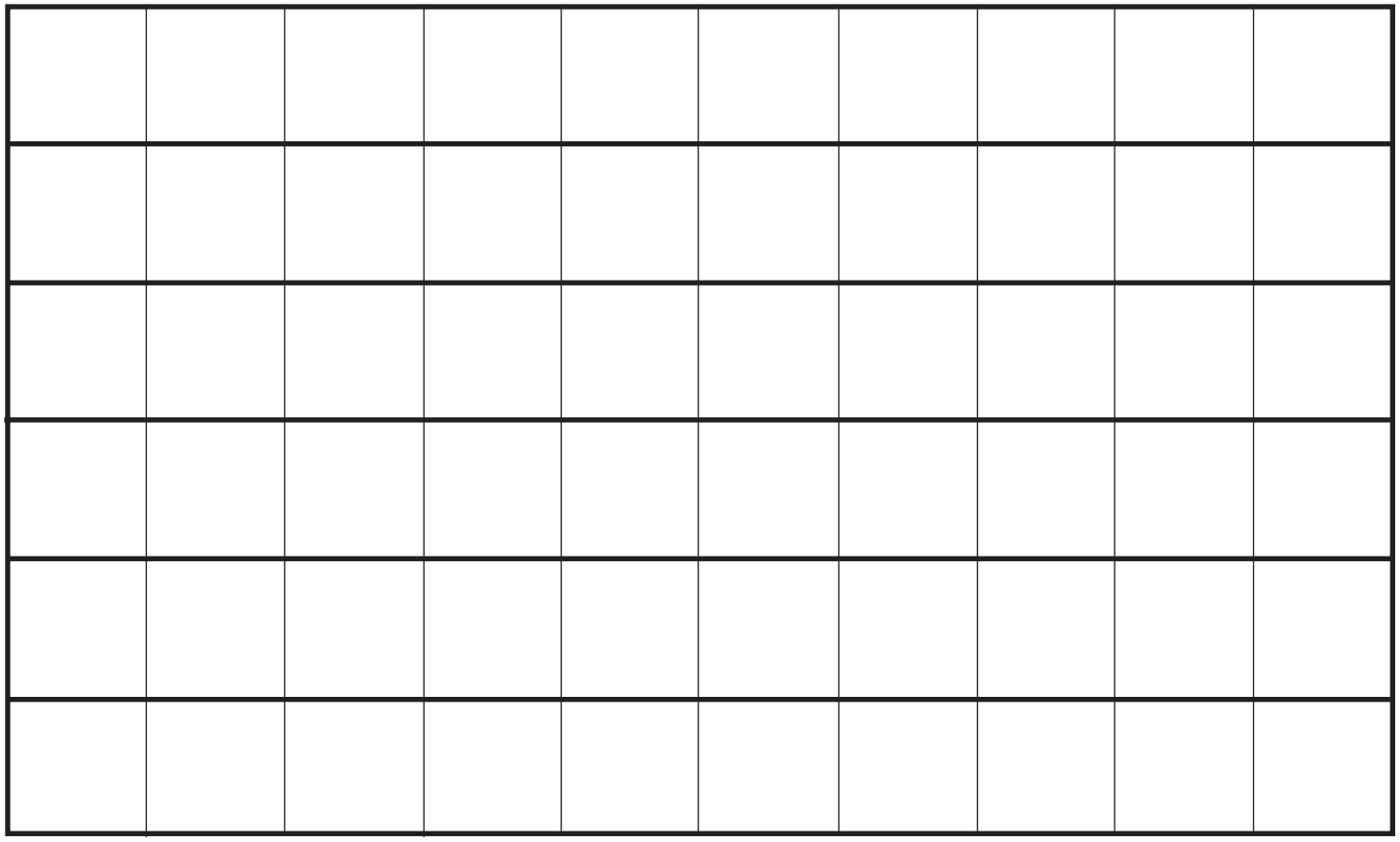
FAST Math +−÷×

10-Stacks

Name: _____

Date: _____

Choose a color for the 10-stacks. Color and cut them out.



FAST Math +−÷×

Name: _____ Date: _____

Numeral Cards (addition)

Print and cut out these cards on heavy paper stock. Use these cards along with the addition and subtraction Numeral Cards to build number sentences and to prompt student navigation along the number line.

+1	+2	+3	+4
+5	+6	+7	+8
+9	+10		

Name: _____ Date: _____

Numeral Cards (subtraction)

Print and cut out these cards on heavy paper stock. Use these cards along with the addition and subtraction Numeral Cards to build number sentences and to prompt student navigation along the number line.

-1	-2	-3	-4
-5	-6	-7	-8
-9	-10		

FAST Math +−÷×

Name: _____ Date: _____

1-100 Number Cards

Print and cut out these cards on heavy paper stock. Use these cards along with the addition and subtraction Numeral Cards to build number sentences and to prompt student navigation along the number line.

1	2	3	4
5	6	7	8
9	10	11	12

Name: _____ Date: _____

1-100 Number Cards

13	14	15	16
17	18	19	20
21	22	23	24

FAST Math +−÷×

Name: _____ Date: _____

1-100 Number Cards

25	26	27	28
29	30	31	32
33	34	35	36

FAST Math +−÷x

Name: _____ Date: _____

1-100 Number Cards

37	38	39	40
41	42	43	44
45	46	47	48

FAST Math +−÷×

Name: _____ Date: _____

1-100 Number Cards

49	50	51	52
53	54	55	56
57	58	59	60

Name: _____ Date: _____

1-100 Number Cards

61	62	63	64
65	66	67	68
69	70	71	72

FAST Math +−÷×

Name: _____ Date: _____

1-100 Number Cards

73	74	75	76
77	78	79	80
81	82	83	84

Name: _____ Date: _____

1-100 Number Cards

85	86	87	88
89	90	91	92
93	94	95	96

FAST Math +−÷×

Name: _____ Date: _____

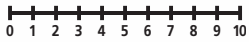
1-100 Number Cards

97	98	99	100

FAST Math +−÷×

The Counting System Intervention

Developing skills for navigating the number system



The lessons in this intervention focus on developing students' robust and flexible number sense. The routines promote a variety of counting skills—counting up; counting on from any given number; counting by 10s, 5s, and 2s; and counting back, to name a few. They are designed to build ease and assurance in navigating our base-ten number system. Well-developed counting skills, both oral counting and counting in conjunction with number lines and number grids, comprise a repertoire of skilled moves on the mental number line—and are crucial to developing students' number sense.

Features of intervention

This math intervention provides instructional features to ignite engagement and propel progress. These include:

- 15 lesson plans with a consistent lesson routine
- Hands-on counting materials
- Multi-sensory instructional techniques
- Repeated practice (frequent, brief, and timed drills)
- Teacher/student dialogue with corrective feedback
- Teacher modeling and the encouragement of students' self-talk
- Progress monitoring

Frequency of intervention

The lessons in the intervention have been carefully crafted to support progress and success. They are designed to include:

- Small group usage (can also be used one-on-one, if necessary)
- 20–30 minute sessions
- 5 days per week over 3 weeks

If five days a week is not feasible, then a minimum of three lessons weekly is important to keep up the learning momentum. (This would extend the intervention to approximately five weeks.)

Lesson routines

All the lessons in the Counting System Intervention are structured in a similar way. Using an exercise workout metaphor, the lessons consist of a Warmup, Workout, and Cooldown. Within each section of the lesson, there are specific routines that you and your students will follow. The routines are explained in detail in Lesson One, which can be used as a reference as you progress through the intervention. These routines will become internalized with use, but here's an overview of the components.

Warmup (5–10 minutes)

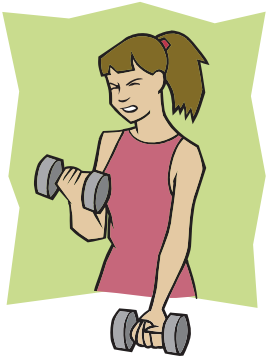
Opening Exercise—a short physical exercise that unifies the class, captures their attention, and starts off the lesson.

Count Together—a choral count aloud involving all students.

Round Robin—students count, one after the other, in a sequence with the teacher changing the counting instructions every 30 seconds or so.

Number Grid—students engage in brief, focused activities with a number grid.

Workout (10–15 minutes)



Talk—a teacher-led discussion about the task to be explored.

Explore—a hands-on exploration of a variety of counting tools such as number grids, number lines, cards, etc.

One-Minute Worksheet—a worksheet of math problems that students work on for one minute with a focus on accuracy versus completion.

Cooldown (5 minutes)

Number Grid—students fill in an empty hundreds number grid.

Reflect—students reflect on what they learned, their successes, and areas of difficulty.

Paperclips—a selected student adds paperclips to a running collection and updates a scorecard to reflect the new total.

Tip:

Elongating lessons doesn't make them more effective. Tightening your use of time will!

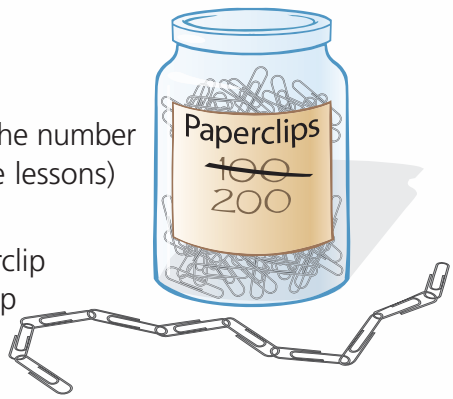
- When an activity drags, move on.
- Be aware of time and push forward to the next routine. A good pace keeps the lesson engaging and fun.
- If students are struggling, repeat the lesson another day instead of elongating the time.
- If need be, go back to practicing routines and transitions so they flow.

Teacher preparation

Before you begin the intervention lessons, there are several things you can do to save you time in the long run. Many of the materials and reproducibles are used throughout the fifteen intervention lessons and can be gathered, constructed, or copied ahead of time. Other materials are specific to certain lessons and are listed among the materials section for each lesson. All reproducibles and lesson plans can be accessed through the PDF files in the software.

Materials needed for all lessons

- 1 Clothespin
- Digital Timer
- Pencils
- Number Grid Folder (1 per student, a place to store the number grids that each student completes at the end of the lessons)
- Paperclips (100 per lesson; 1,500 total)
- Jar, box, or other container (used to hold the paperclip collection. Attach a scorecard to the outside to keep a running tab of the total.)



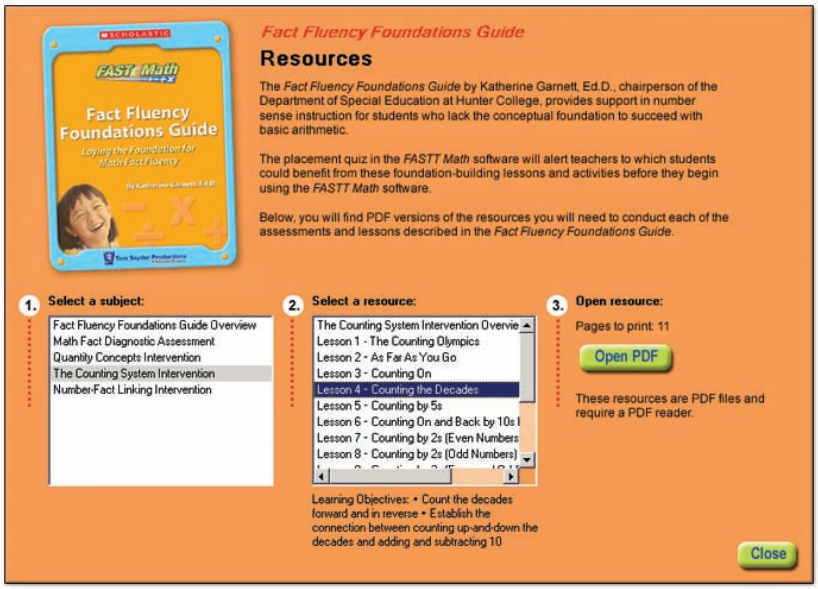
Print and assemble:

These PDFs are used during most lessons in the Counting System Intervention. Print them in Lesson 1 and keep them on hand throughout the unit.

- 3-Part Lesson Chart (1 for the group, print and laminate)
- 0–300 Number Lines (1 for the classroom, 1 for each student; use the template to create your own number lines, if needed)
- Blank 100 Number Grids (1 for each student per day, at least 15 total per student)
- Filled-in 100 Number Grids (at least 15 total per student)

Accessing the PDF files

The PDF files for each lesson can be accessed through the teacher administration tool (see page 4 for instructions). You'll find a PDF for each lesson, complete with lesson instructions and all the blackline masters you will need. The reproducible blackline masters are only available through the PDF files.



Lesson worksheets are easily available as PDFs through the teacher administration tool.

Notes:

- See page iv for information about the teacher administration tool.
- In *FASTT Math Enterprise Edition*, you can navigate to each section by clicking a bookmark after you open the PDF file of the guide.

Lesson 1:

The Counting Olympics

Learning objectives

- Understand the Counting Olympics theme in a way that feels age appropriate (i.e., Dispel any sense that this is "baby stuff.")
- Orient students to the instructional routines, transitions, and materials that will become the daily lesson structure

This first lesson focuses on becoming familiar with the lesson parts, instructional routines, and transitions used in subsequent counting lessons. It does not aim to "teach" math content, but rather to familiarize students with the lesson format. The lesson aims to make the routines explicit, providing tips about what to say and do to help students become familiar with the process. Practicing the routines until they flow easily will make the lessons both effective and fun.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 1:
  - 3-Part Lesson Chart
  - 0–300 Number Line
  - Blank 100 Number Grid (1 for each student)
  - One-Minute Worksheet (1 for each student)
  - Filled-in 100 Number Grid (1 for you and 1 for each student)
- Copy the PDFs so that there are enough copies for each student

Preparing students

Understand the Counting Olympics theme in a way that feels age appropriate.

1. Describe to your students what you'll be doing in this lesson.
We're going into training for The Counting Olympics. Olympic Training means practice, practice, practice. It also means focusing, teamwork, and fun.

2. Discuss briefly what Olympic teams do (run, jump, swim, gymnastics, etc.) and how they train: practice, practice, practice; focus, teamwork, fun.

- *So, if we're training for The Counting Olympics, what are we going to do?* (*lots of* ***counting*** *practice, practice, practice; focusing, teamwork, and fun*)
- *We're going to exercise our math brains so they become stronger and faster.*
- *So now you tell me. What are we in training for? What is this class called?* (*The Counting Olympics*)
- *And what do athletes do when they're in training?* (*practice, etc.*)
- *And why are we doing this?* (*to become stronger, faster math thinkers*)

Orient to the routines and materials that will be the daily structure.

3. Provide students with an overview of what to expect. Explain that each day's lesson will follow the same process of Warmup, Workout, and Cooldown. Make the following points:

- Every training has three parts: Warmup, Workout, and Cooldown, each with subparts. (show the three-part lesson chart).
- Every day someone is in charge of moving the clothespin and calling out the lesson part as we move from warmups to workouts to cooldowns. AND, that person gets to be the paperclip master at the end of the lesson. (Choose a student for today.)

Warmup

(5–10 minutes)

1. **Student of the day:** Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup*." That is the cue for everyone to start the opening exercise.

2. Explain to students that, "*In this warmup, we do an Opening Exercise, Count Together, do a Round Robin, and do Number Grids*."

Opening Exercise

3. Have your students choose a physical routine to accompany a very brief opening count-together. Be sure to choose a physical routine and practice it. The point of starting out with something physical and in unison is to capture attention and start every lesson together.

Opening exercise suggestions to accompany brief choral counting:

- raise arms up-down
- finger calisthenics
- jogging rhythmically in place
- fingertips touch shoulders, arms up

Practice the opening exercise together with a very brief counting warmup, having everyone count in unison and at a pace that keeps everyone together. Start out counting from 1–10 forward and then 10–1 back.

Note:

Counting Sequences
In future lessons, as appropriate, replace this easy sequence with harder ones, such as 10–20 forward/20–10 back, 50–60/60–50, and 90–100/100–90.

Reminder!

The purpose of Lesson 1 is to get familiar with the Counting System lesson routines (setting out materials, how to do the activities, putting away, and transitioning to the next lesson part), so the math content below is "lite."

Count Together

4. Lead this choral count-aloud with students, setting the pace and re-counting at any wobble spots. This is to help the students "catch" the repeating sequences of the number system by ear. Some physical accompaniment can help (leading an orchestra or pumping fists up and down to set the beat). For this first Count Together, count up to 50.

5. Reinforce what was done.
"Great! OK, so in our warmups we do a physical routine and we count-up-and-up together. You got it!"

Round Robin

6. Explain how a round robin will be done in every lesson: Students count one after the other in sequence; the teacher interrupts the counting with a HALT gesture and provides new counting instructions (e.g., *start with__, now count by__, reverse counting direction,* etc.); the round robin counting continues, following the new instructions.

7. First, practice brief straightforward round robin counting (say, 1–15). Round robins are helped by a steady rhythm, so orchestrate as needed. When students demonstrate that they can pay attention and "go-around," praise their work and let them know that now you will make it more challenging.

8. Next, practice round robin counting again from 1, with the insertion of your HALT gesture and your change of instructions, such as:
HALT: Start over at 5.
HALT: Reverse the direction of the round robin.
HALT: Reverse the direction again.
HALT: Start with 10 and count backwards.
HALT: Start with 20 and count backwards.

After a successful round robin, give students positive feedback:
"Now you know how to do warmups—our opening routine, counting together, and round robin! And everyone is paying attention! This is powerful teamwork!"

Number Grid

9. Introduce number grid work using the Filled-in 1–100 Number Grid. Give students an easy task (like having them circle all the numerals containing zeros), so they can get the idea of how to work with a number grid.
"Great, now you know that in warmups we also do number grids!"
"So, what's next after warming up?" (*We workout.*)

10. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

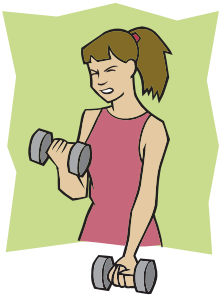
Transition to Workout

Below are ideas for both transitioning to Workout and establishing the instructional routines. Specifying plans, these or your own variations, is crucial so habits form that keep the lessons moving forward.

- **Announcing and time-keeping**
  Cue the student of the day to move the clothespin to the *Workout* icon and call out "*Time for Workout—45-second transition.*" Student sets the timer for 45 seconds and announces, at the right point, "*Last ten seconds—10, 9, 8, 7, 6, 5, 4, 3, 2, Workout!*" Of course, everything is in place and everyone is ready. (If not, practice it again.)

- **Preparing materials**
  Have students set out materials for both *Workout* and *Cooldown* in the 45-second transition: Specify who does what in this transition, including specific places that materials are set (number lines, number grids, pencils, and math tool bags). If useful, you could even provide mats with silhouettes for where "tools" go. Consider having everyone move during this timed transition, each completing a simple materials job and then changing seats. Permitting a bit of noise and chaos on the way to resettling can make for energized attention once they complete the transition.

- **Musical accompaniment**
  Have everyone hum a tune while shifting materials.



Workout
(10–15 minutes)

1. Remind students, "*Remember, when we do our workout, we talk about things, we explore things, and we do a One-Minute Worksheet.*"

Talk: Favorite Numbers and A Googol

2. Lead a brief discussion about the following (write responses on the board, overhead, or chart pad):
   - *What students' favorite numbers are and why…*
   - *What number is written with a…*
     - *1, followed by 1 zero (answer: 10)*
     - *1, followed by 2 zeros (answer: 100)*
     - *1, followed by 3 zeros (answer: 1,000)*
     - *1, followed by 4 zeros (answer: 10,000)*
     - *1, followed by 5 zeros (answer: 100,000)*
     - *1, followed by 6 zeros (answer: 1,000,000)*
     - *1, followed by 100 zeros (answer: one googol)*

3. Have the students help you write a googol. You can do the writing and have students count the zeros in unison as you write. Or you can have ten students come to the board and write ten zeros each. Then, the class can count the zeros and verify that they've created a googol.

   "OK, that was the talk part of our workout! That was a thoughtful discussion, with everybody really listening."

Explore: Hundred Grid

4. Explain that you're going to explore the 100 Number Grid that they circled in the Warmup. Pose simple exploratory questions, such as:

- *Where is the fifth row?*
- *Where's the fifth column?*
- *How many numerals are in the second row?*
- *How many in the last row?*
- *How many numerals are in the third column?*
- *How many numerals in the last five columns?*
  (*You could further explore row vs. column, numeral vs. number*)

"*Now that was the exploring part in our workout!*"

One-Minute Worksheet

5. Explain that the workout usually ends with a one-minute page of problems. Their job is to finish as many of these as they can correctly in one minute.

6. Provide the One-Minute Worksheet and time students for 60 seconds.

7. Remind the students, "*So, now we've done our workout—we talked, we explored, and we did the One-Minute Worksheet!*" Then have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown

Choose and practice a simple, specific transition routine, for example:

- Cue the student of the day to move the clothespin on the lesson chart and call out, "*Time for Cooldown!*"
- Have students hum a tune while shifting materials. (There should be no need to get up.)
- Do something physical and in unison to end the transition—perhaps rubbing hands to count of 3, snapping fingers to count of 3, and then sighing together in a whisper, "*Ready!*"

Cooldown

(5 minutes)

1. Explain to students what to expect in the Cooldown:
   "To cooldown, everyone gets quiet. We each take a Blank 100 Number Grid, and each person fills it in quietly. Then, we reflect quietly together. Then we add 100 paperclips to our jar."

Number Grid

2. Have each student fill in a Blank 100 Number Grid from 1 to 100 (or less if it's too difficult). Anyone who doesn't complete it now should do so before the next lesson, because they'll have it to use then.

Note:

As students proceed through subsequent counting lessons, they will be creating a collection of Filled-in 100 Number Grids through 1000 (or higher). Establish a place to store these safely as they accumulate from lesson to lesson. They could, for example, be kept in a class number grid folder, with each student's collection in a binder clip.

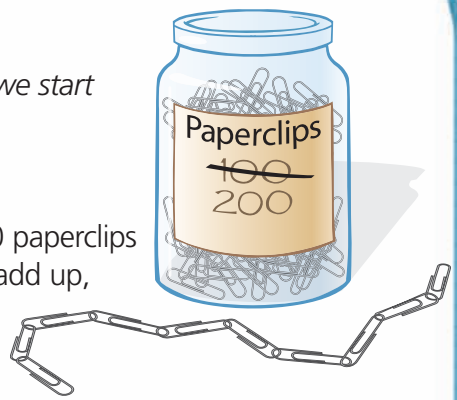
"So, we are really cooling down. We quietly filled in our 100 grid, now we quietly reflect."

Reflect

3. Practice a very brief reflection, so students understand how the Cooldown proceeds. Possible reflection topics include:
   - the biggest number students learned today
   - which part of the Counting Olympics seems like the most fun
   - which part they think they will learn the most from

Paperclips

"We're at the end of our Counting Olympics practice, so now we start our paperclip collection with our first one hundred paperclips."



The Great, Ongoing, Accumulating Paperclip Collection

This is a daily session-ending routine in which students add 100 paperclips (ten strings of ten) to a running collection and watch the total add up, hundred by hundred.

Prepare the materials ahead of time:

1. As an activity outside of these lessons, have students make several strings of ten paperclips, turning a box into a "Paperclip String Factory."
2. Dedicate a jar, box, or some kind of special container to hold the paperclip collection as you add 100 paperclips at the end of each lesson.
3. Attach some kind of scorecard to the outside of the container for students to use to keep a running tab on the total number of paperclips they accumulate.

Once the materials are prepared:

4. Explain to students how the paperclip collection will work for every lesson. *"The student of the day will draw ten strings of paperclips from the Paperclip String Factory, add them to the paperclip collection container, and then write down the new grand total."*
5. Accompany this ritual by an exchange in which you verbalize each aspect and ask related questions:
   - *So, remind us, how many paperclips did we use to make each string?*
   - *And how many **strings** are we adding to our jar?*
   - *So, how many **more** paperclips are we adding today?*
   - *So, now how many paperclips do we have in our jar?*
6. Have the student of the day write +100 on the running tab on the outside of the container and calculate the new grand total.
7. Let everyone participate in some fun finale (like a silent cheer or finger calisthenics countdown) and conclude the lesson.

3-Part Lesson Chart

Warmup
(5-10 minutes)

- Opening Exercises
- Count Together
- Round Robin
- Number Grid

Workout
(10-15 minutes)

- Talk
- Explore
- One-Minute Worksheet

Cooldown
(5 minutes)

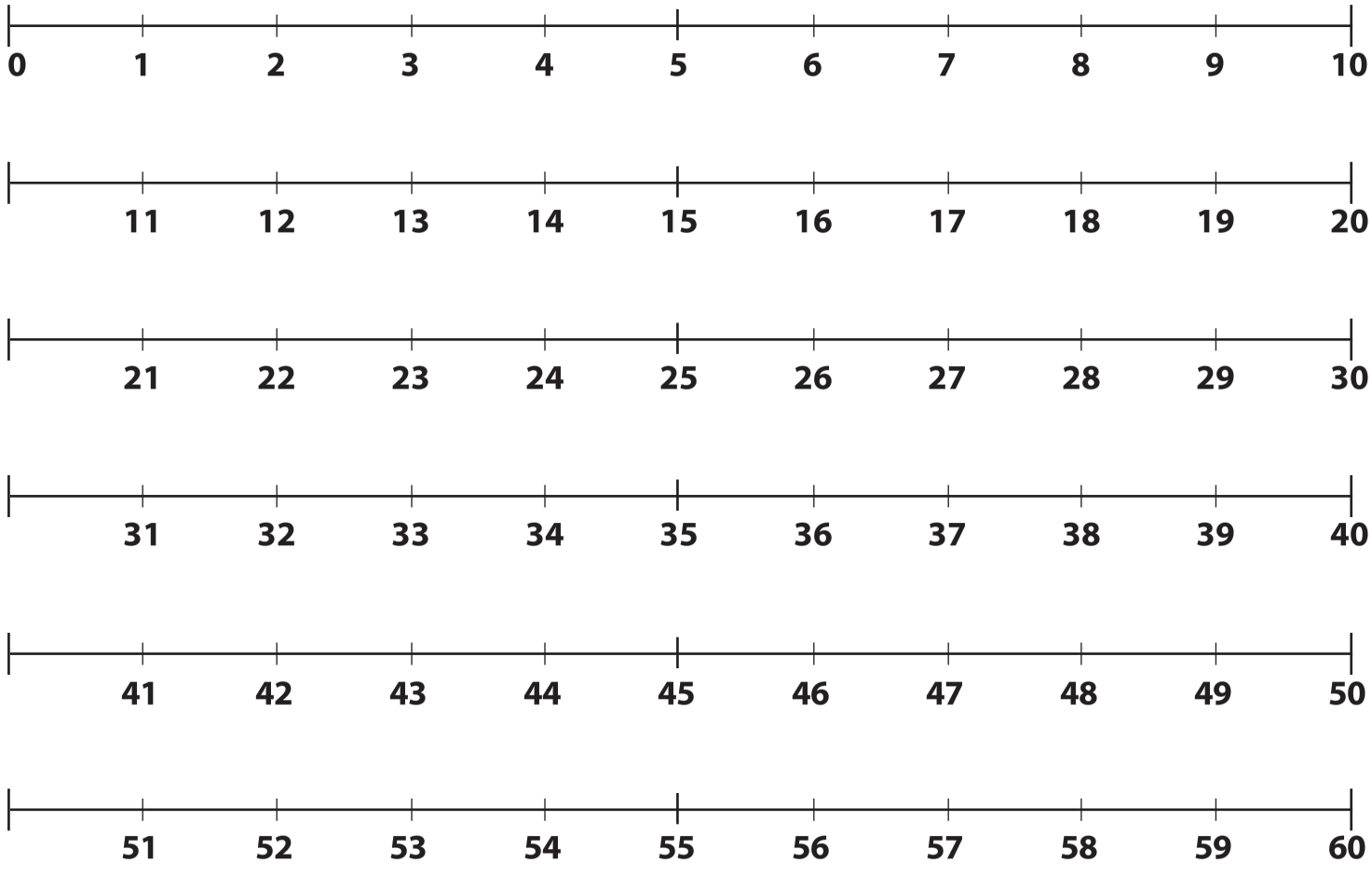
- Number Grid
- Reflect
- Paperclips

FAST Math +−÷x

Number Line: 0-60

Name: _____

Date: _____

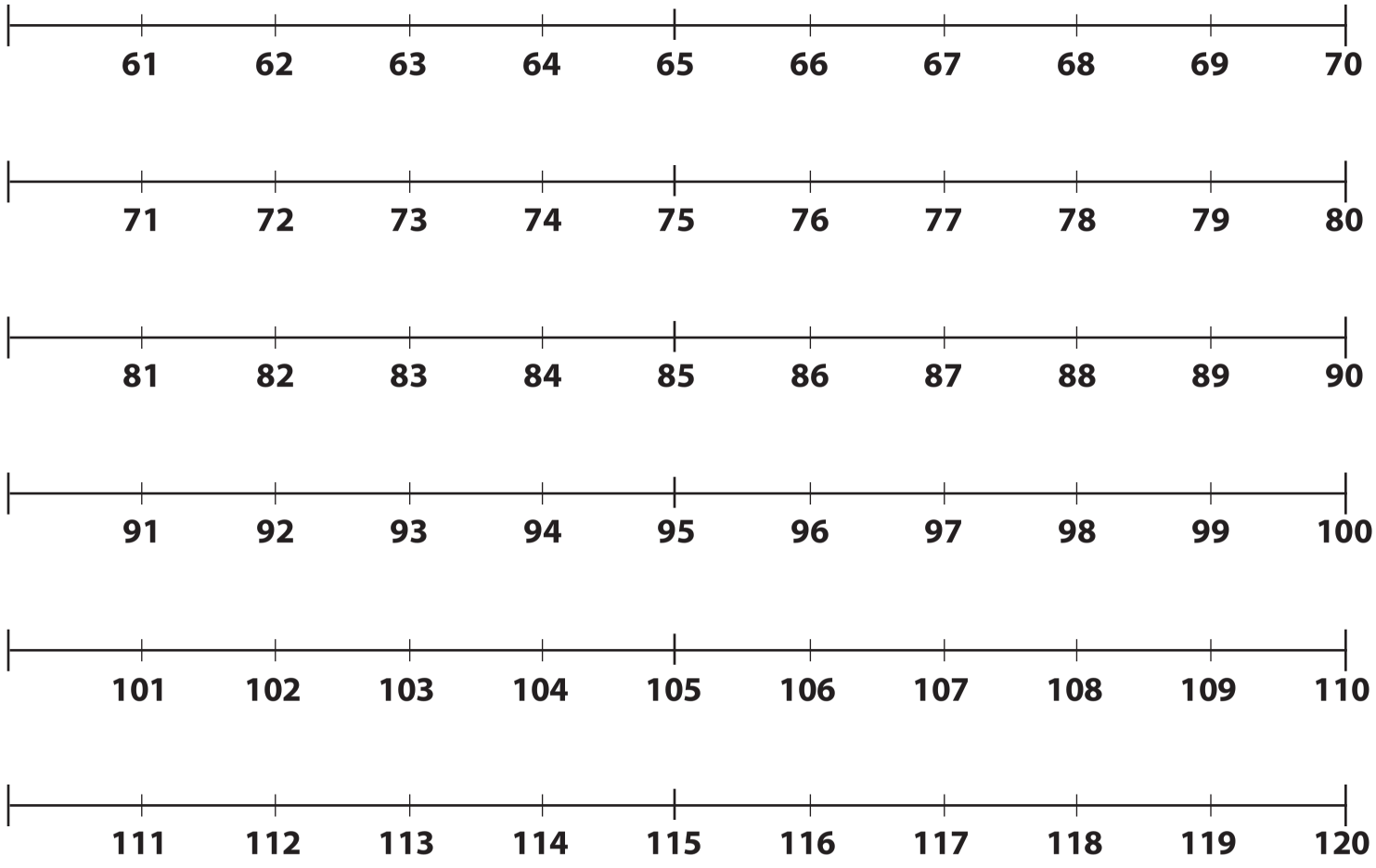


FAST Math +−÷×

Number Line: 61-120

Name: _____

Date: _____

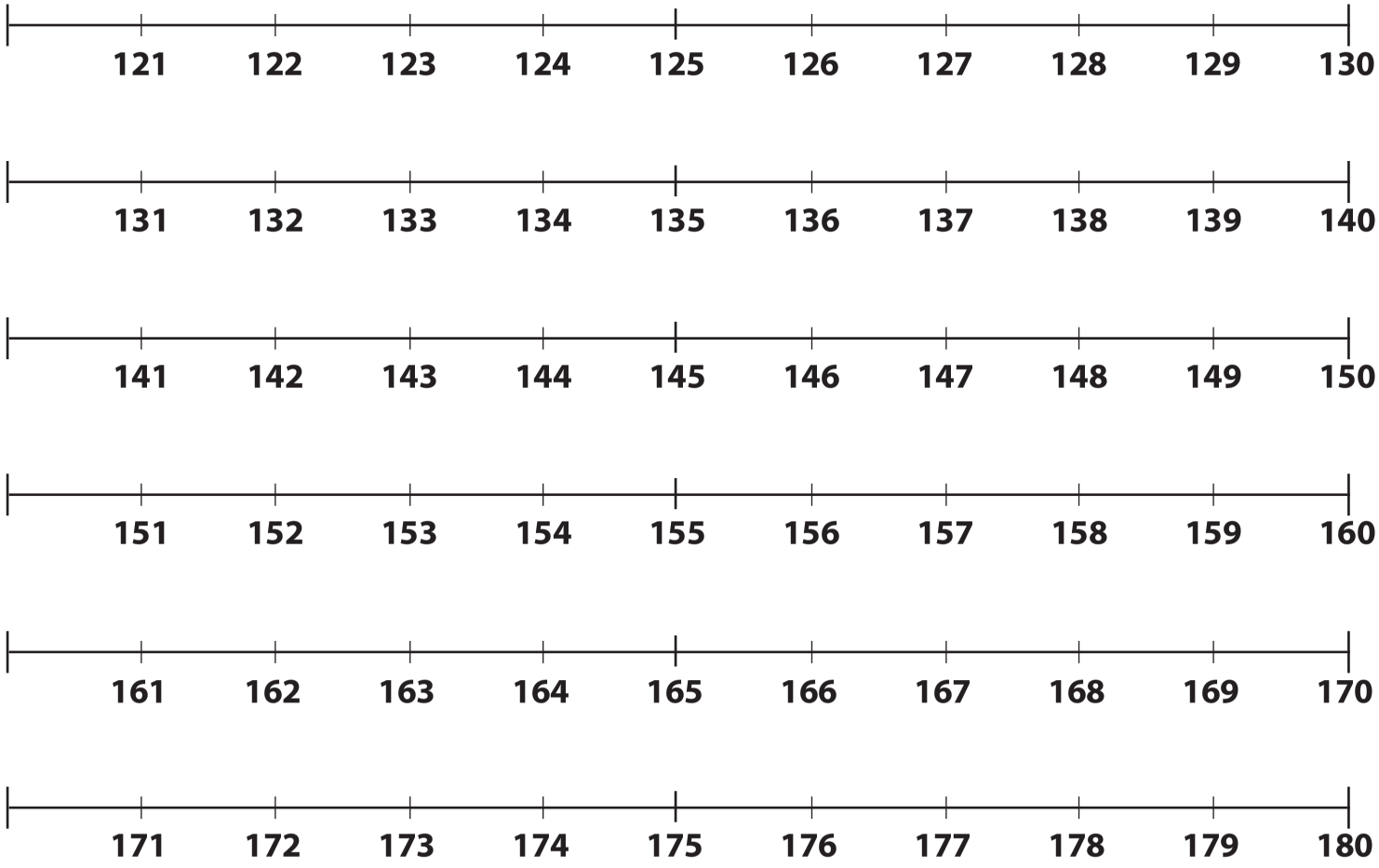


FAST Math +−÷x

Number Line: 121-180

Name: _____

Date: _____

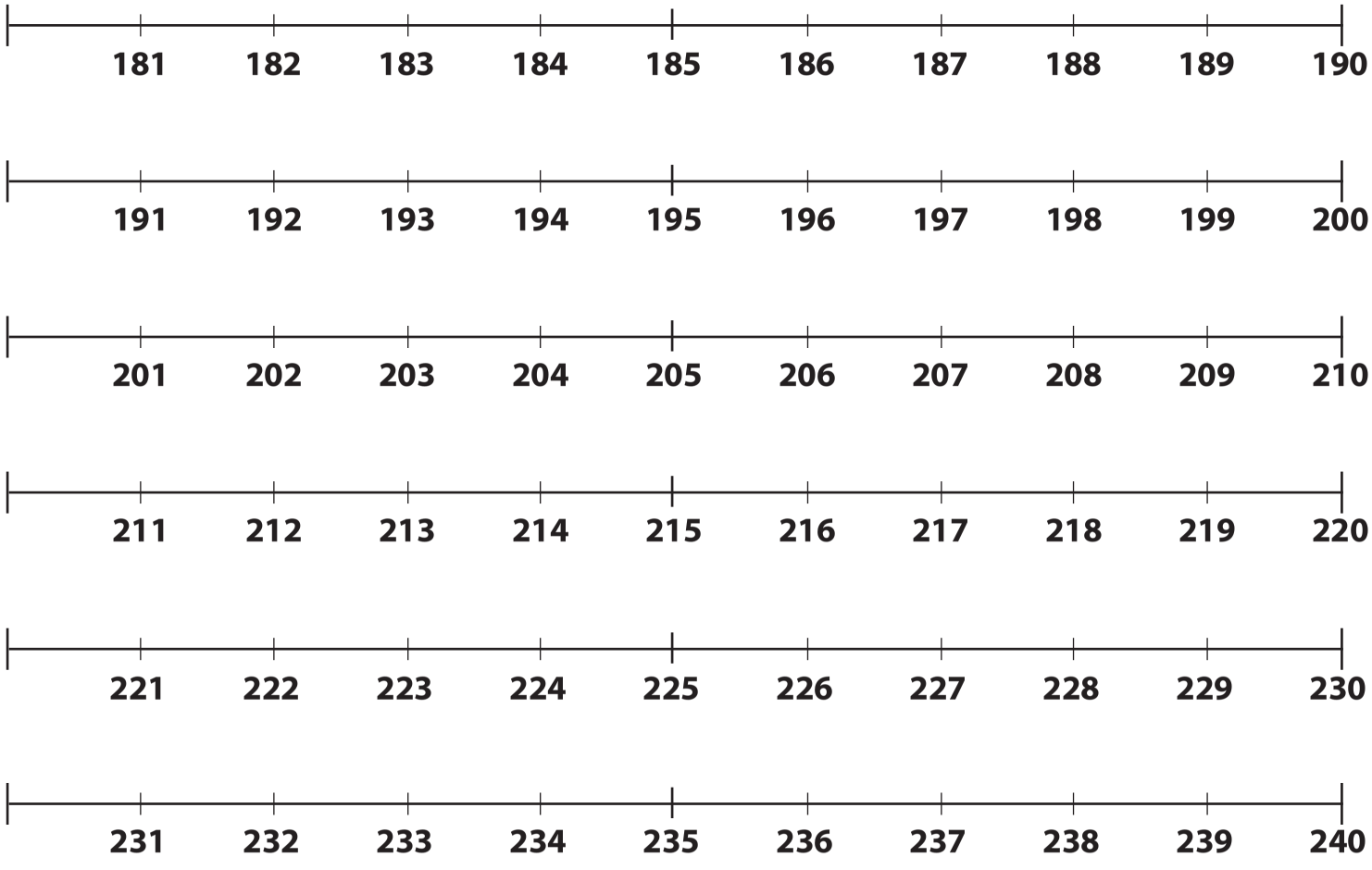


FAST Math +−÷×

Number Line: 181-240

Name: _____

Date: _____

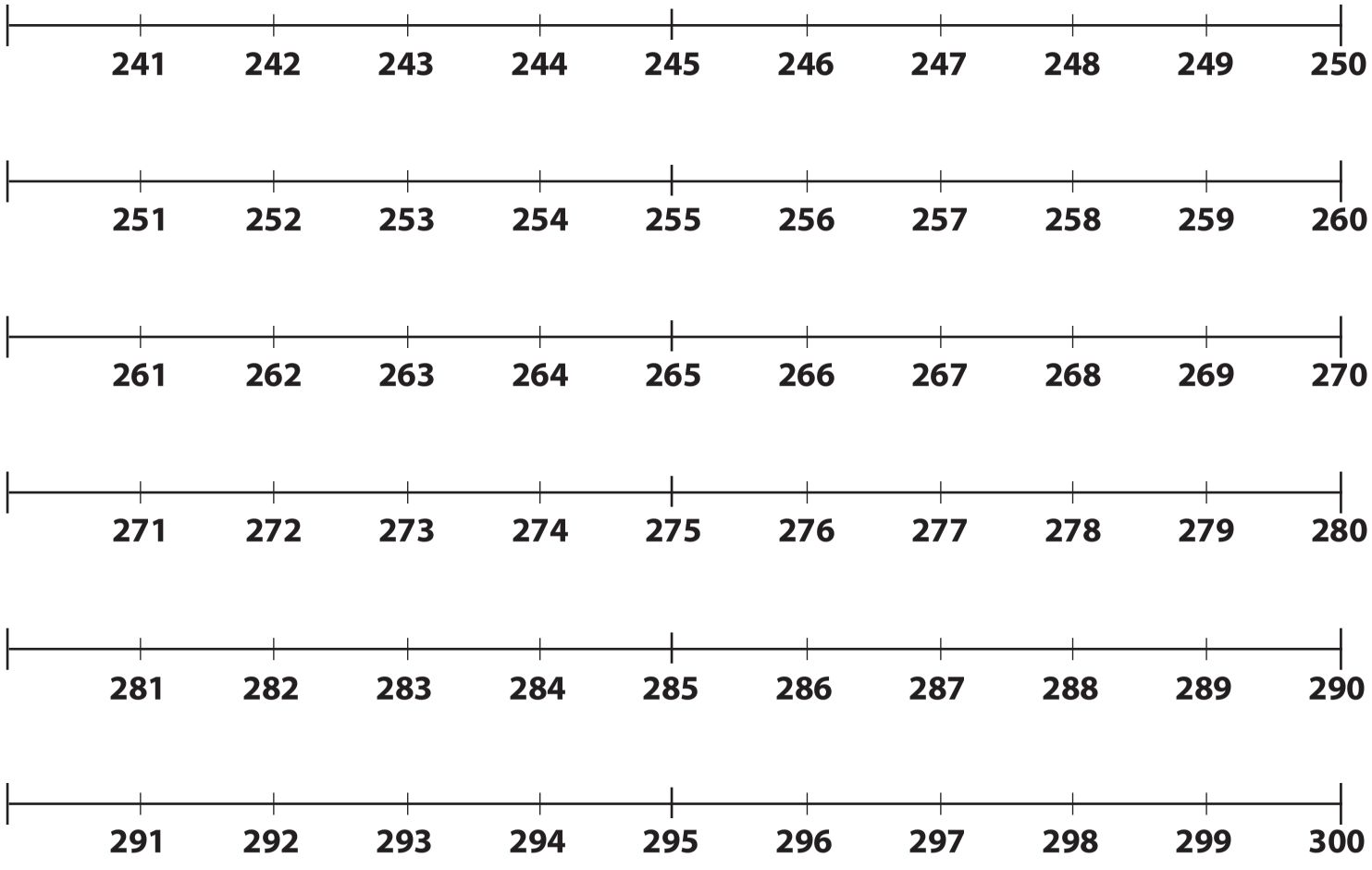


FAST Math +−÷×

Number Line: 241-300

Name: _____

Date: _____



FAST Math +−÷×

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Lesson 1: One-Minute Worksheet

5+3=	4+4=	2+2=	4 +1	2 +4	3 +5
4+2=	3+2=	2 +1	2 +5	1 +2	2+4=
5 +5	5 +2	3+5=	4+3=	4 +3	5 +4
5+2=	3+3=	3 +5	4 +5	2+1=	2+5=
4 +2	4 +1	1+3=	5+5=	4 +5	2 +2
1 +1	5 +4	3 +4	5 +5	5+5=	2+2=
4 +4	1+4=	5+4=	3 +3	5+3=	2 +1
4+3=	3 +1	2+3=	1 +5	3+5=	5+3=
3 +4	1 +4	4+4=	3+5=	2 +3	3 +4
1+1=	3+4=	1+5=	2 +5	5+1=	3+4=

FAST Math +−÷x

Lesson 2:

As Far as You Can Go

Learning objectives

- Strengthen rote counting and extend how far students can count
- Reinforce counting at wobble and breakdown points

Rote counting is not mindless; it reflects learners' early understanding of our base-ten system. Counting aloud with your students can reveal to them the system of our numbers, helping them hear its logic. Children need to catch on to this system with their mind's ear through counting aloud and then connecting that to a written number line. Focus counting practice at breakdown points and wherever performance wobbles—then stretch how far students can count.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 2:
  - Number Squares 1–100 (1 set of cut-up squares for each pair of students)
  - Vertical Decade Chart (1 for group, or create own on the chalkboard)
  - Filled-in 100 Number Grid (1 for each student)
  - Missing Numbers 1–100 Grid (1 for each student)
  - Blank 100 Number Grid (1 for each student)
  - One-Minute Worksheet (1 for each student)
  - Number Crisscross Game Card (1 for the group)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

2. Remind students, "*In our warmup, we do our opening exercise, we count together, do a round robin, and then number grids.*"

Opening Exercise

3. Do the 30-second physical Opening Exercise you chose from the suggestions in Lesson 1.

Count Together

4. Lead a choral count-aloud by 1s, pointing to—and emphasizing—each decade displayed in the decade chart (or on the chalkboard). Count together as high as makes sense (50? 100? 200?). Practice the parts that are still troublesome to students.

Variations:

- Try putting the stress on the decade-word—*thirty*-one, *thirty*-two
- To strengthen engagement, accompany counting with physical routines (e.g., Wiggle finger per digit, clap on the decade).

5. Reinforce what students have done.

10
20
30
40
50
60
70
80
90

Round Robin

6. Conduct the Round Robin as described in Lesson 1. Begin the Round Robin counting by 1s from wherever seems suitable (e.g., from 1, from 20, from 53). Interrupt the Round Robin with a HALT gesture, calling out re-directions, such as:

- *Count backwards from there*
- *Count forwards again*
- *Start over at 50*
- *Reverse relay direction*

How high you go depends upon your students. This is just a warmup, so do not make it laborious. Keep it brief, peppy, and diagnostic.

7. Reinforce what students have done and their teamwork.

Number Grid

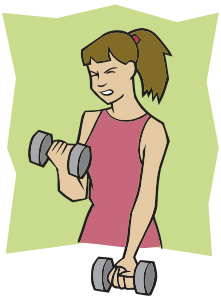
8. Give students one minute to fill in the Missing Numbers 1–100 Grid, coaching any who need help.

9. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Note:

Do not let this take more than about one minute. It is not important that all students finish this; it is important that the instructional pace not get bogged down.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout
(10–15 minutes)

Talk: Number Crisscross Game

1. Talk through and demonstrate to the group how the game is played. Then have students play it.

Number Crisscross Game

Number of Players: 2-3

Materials:
- **Blank 100 Number Grid**
- **Filled-in 100 Number**

Start by cutting up the Filled-in 100 Number Grid worksheet. Then choose 10 number squares at random and place them in their correct positions on the Blank 100 Number Grid worksheet. Divide up the remaining squares between players (squares remain face down). Without looking, a player chooses one square to place on the grid. A player can only place a square before, after, above, or below a number already on the grid (crisscross). Players who cannot place their square must pass and that square is again placed down in front of the player. The game continues until a player wins by running out of squares.

Explore: Your Number Line

2. Lead an exploration of students' 0–300 Number Lines. Have students explore using their number lines as measuring tapes, measuring various items and body parts.

3. Have students further explore their number lines by posing the following questions and having them use their individual number lines to answer:
- *What's the lowest number?* [0]
- *What's the highest number?* [300]
- *Your number line is folded. What are the numbers it is folded at?*
- *Fold the number line in half. What's in the center?* [150]
- *What number would come after the last number?* [301]
- *What could you use your number line for?* [Accept all reasonable answers.]

One-Minute Worksheet

4. Provide the One-Minute Worksheet and time students for 60 seconds.

5. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 1 to 100. Add each student's completed grid to their Number Grid Folder.

Reflect

2. Lead a short discussion with students. Ask them:

- *How high did you count?*
- *What part was easy and where did you have to "think hard"?*
- *How high would you have to count to count all the people in the world? all the stars in the sky?*

Note:

Students may need modeling and explanation to effectively reflect on and verbalize their learning.

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Vertical Decade Chart

10
20
30
40
50
60
70
80
90

FAST Math +−÷x

Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

Name: _____ Date: _____

Missing Numbers 1-100 Grid

Fill in the missing numbers on the grid below.

1	2	3		5	6	7		9	10
11		13	14		16	17	18		20
	22	23	24	25		27	28	29	
31		33		35	36	37		39	40
41	42	43	44		46		48		50
	52		54	55	56	57		59	60
61	62	63		65		67	68		70
71		73	74		76	77	78	79	
81	82	83	84	85		87		89	90
91		93		95	96		98	99	100

FAST Math +−÷×

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Lesson 2: One-Minute Worksheet

1 +2	3 +4	5+1=	2+3=	5 +5	5 +4
1+1=	4+3=	2 +2	1 +5	4+4=	3+5=
3 +1	5 +4	1+4=	3+3=	4 +3	4 +1
2+4=	3+5=	5 +2	4 +5	2+5=	5+3=
4 +2	4 +1	1+3=	5+5=	3 +5	2 +5
3+2=	5+3=	2 +4	4 +4	1+5=	2+2=
1 +1	2 +5	5+4=	2+1=	3 +4	2 +1
5+2=	2+2=	3 +5	1 +4	5+3=	3+4=
5 +5	2 +1	4+4=	3+5=	2 +3	3 +4
5+5=	3+4=	3 +3	4 +5	4+2=	4+3=

FAST Math +−÷×

Name: _____ Date: _____

Number Crisscross Game Card

Follow the instructions on the game card below to play Number Crisscross.

Number Crisscross Game

Number of Players: **2-3**

Materials:
- **Blank 100 Number Grid**
- **Filled-in 100 Number Grid**

Start by cutting up the Filled-in 100 Number Grid worksheet. Then choose 10 number squares at random and place them in their correct positions on the Blank 100 Number Grid worksheet. Divide up the remaining squares between players (squares remain face down). Without looking, a player chooses one square to place on the grid. A player can only place a square before, after, above, or below a number already on the grid (crisscross). Players who cannot place their square must pass and that square is again placed down in front of the player. The game continues until a player wins by running out of squares.

FAST Math +−÷×

Lesson 3:

Counting-On

Learning objectives

- Count forward and in reverse by 1s from different starting points
- Establish the connection between counting forward and backwards by 1s and adding and subtracting 1.

Counting-on is a significant developmental step. Counting-back is more difficult, mirroring the greater difficulty of subtraction over addition. Exercising our students' counting-back skills is valuable, but we should not expect the same ease and speed as with counting forward.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather materials specific to this lesson:
  - Number Squares 1–100 (from Lesson 2)
- Print the PDFs for Lesson 3:
  - Vertical Decade Chart (1 for each group)
  - +1 and -1 Card Pack (1 for each group)
  - Missing Numbers 1–100 Grid (1 for each student)
  - Blank 100 Number Grid (1 for each student)
  - Filled-in 100 Number Grid (1 for you and 1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup*."

2. Remind students of what they do in the Warmup.

Opening Exercise

3. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

4. Lead a choral count-aloud by 1s, pointing to—and emphasizing—each decade displayed in the decade chart. Decide on an appropriate starting point and count as high as makes sense for your students. Practice the parts that are still troublesome to students.

10
20
30
40
50
60
70
80
90

Round Robin

5. Conduct the Round Robin as described in Lesson 1. Begin the Round Robin counting by 1s from wherever seems suitable (e.g., from 20, 53, 192). Interrupt the Round Robin with a HALT gesture, calling out re-directions, such as:

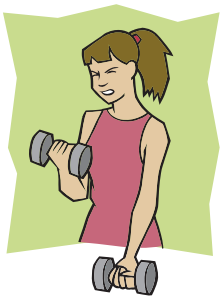
- *Count backwards from there*
- *Count forwards again*
- *Start over at 50*
- *Reverse relay direction*

Number Grid

6. Give students one minute to fill in the Missing Numbers 1–100 Grid, coaching any who need help.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: + and -1

1. Explain that counting by 1s is the same as adding 1. Model adding and counting for students, then ask them to think of and share their own examples.

Variations:

Use more challenging examples (say 30–40 or 85–95 or 1,020–1,030), if appropriate for your particular students.

- *Let's count together from 10–20 (proceed to do so)*
- *So, now you know that 10 plus 1 is ___*
- *And 11 plus 1 is ___*
- *And 12 plus 1 is ___*
- *And 19 plus 1 is ___*

2. Explain that counting backwards by 1s is the same as subtracting 1. Model subtracting and counting for students, then ask them to think of and share their own examples.

Variations:

Use more challenging examples (say 40–30 or 95–85 or 1,030–1,020), if appropriate for your particular students.

- *Let's count together from 20–10 (proceed to do so)*
- *So, now you know that 20 minus 1 is ___*
- *And 19 minus 1 is ___*
- *And 18 minus 1 is ___*
- *And 17 minus 1 is ___*
- *And 9 minus 1 is ___*

3. Talk together about the different words for "+" (plus 1, 1 more, add 1, adding 1, addition) and the different words for "-" (minus 1, 1 less, take away 1, subtract 1, subtracting 1, subtraction).

4. Show and read cards from the +1 and -1 Card Pack. On the Filled-in 100 Number Grid, explain and show how to move forward or back 1, depending upon the problem on the card.

5. Have students read and answer +1 and -1 problems from the cards, using the different vocabulary words and demonstrating + and - 1 on the Filled-in 100 Number Grid.

Explore: Neighbors on the Number Grid

6. Cover your 100 Number Grid with the matching Number Squares. Then turn the Number Squares over so the blank side is showing.

7. Have students turn their individual 100 Number Grids to the blank side. "*Let's explore our number grids—but, without seeing the numbers!*"

8. Turn over the #3 Number Square and have the students place #3 Number Squares on their individual 100 Number Grids. Then have students find the number that is "1 less" and "1 more" than 3 and place these on their grids, using all the different + and - 1 terminology discussed above (plus 1, 1 more, add 1, adding 1, addition; and minus 1, 1 less, take away 1, subtract 1, subtracting 1, subtraction).

9. Continue using other starting numbers on the 100 Grid: 13, 23, 33, 44, 54, 64, 75, 85, 95.

One-Minute Worksheet

10. Provide the One-Minute Worksheet and time students for 60 seconds.

11. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown
(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 101 to 200. Add each student's completed grid to their Number Grid Folder.

Reflect

2. Reflect with the students about + and - 1 and about all the vocabulary for + and - 1.

3. Lead a short discussion about filling in the Number Grid from 101 up.
   - *What was difficult?*
   - *What was a "strategy" they used to help them?*

Paperclips

4. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Vertical Decade Chart

10
20
30
40
50
60
70
80
90

FAST Math +−÷x

+1 and -1 Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

4+1=	7+1=	3-1=	9-1=	13-1=	16-1=
19-1=	22+1=	25-1=	28+1=	28-1=	30+1=
33-1=	37+1=	37-1=	45-1=	45+1=	42+1=
51-1=	58+1=	58-1=	59-1=	63-1=	67+1=
70+1=	78-1=	78+1=	72+1=	80+1=	86-1=
94+1=	94-1=	99-1=	10-1=	20-1=	30-1=
40-1=	50-1=	60-1=	70-1=	80-1=	90-1=
100-1=	9+1=	19+1=	29+1=	39+1=	49+1=
59+1=	69+1=	79+1=	89+1=	99+1=	

FAST Math +−÷x

Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

Name: _____ Date: _____

Missing Numbers 1-100 Grid

Fill in the missing numbers on the grid below.

1	2	3		5	6	7		9	10
11		13	14		16	17	18		20
	22	23	24	25		27	28	29	
31		33		35	36	37		39	40
41	42	43	44		46		48		50
	52		54	55	56	57		59	60
61	62	63		65		67	68		70
71		73	74		76	77	78	79	
81	82	83	84	85		87		89	90
91		93		95	96		98	99	100

FAST Math +−÷×

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Lesson 3: One-Minute Worksheet

9 -1	7 +1	2-1=	5+1=	1 +1	5 -1
12-1=	10-1=	17 +1	17 -1	23-1=	17+1=
10 -1	23 +1	21-1=	11-1=	31 +1	27 -1
19+1=	31-1=	29 +1	28 -1	22-1=	31+1=
28 +1	37 -1	33+1=	22+1=	52 -1	65 +1
47+1=	44-1=	43 +1	56 -1	53+1=	49+1=
40 -1	69 +1	99-1=	100-1=	89 +1	59 +1
59-1=	70-1=	81 -1	90 -1	69+1=	60-1=
94 +1	74 -1	74+1=	79+1=	91 -1	89 -1
80-1=	80+1=	79 +1	99 +1	50-1=	100-1=

FAST Math +−÷x

Lesson 4:

Counting the Decades

Learning objectives

- Count the decades forward and in reverse
- Establish the connection between counting up and down the decades and adding and subtracting 10

Since decades are the structural units of the base-ten number system, being able to easily count up and down the decades makes other counting easier. If your students are already nimble at counting the decades from 10–100, take them further—to 200 or 300 or higher.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather the materials specific to this lesson:
  - Number Squares 1–100 (from Lesson 2)
- Print the PDFs for Lesson 4:
  - Filled-in 100 Number Grid (with each multiple of 10 circled, 1 for the group)
  - +10 and -10 Card Pack (1 per group)
  - Missing Numbers 101–200 Grid (1 for each student)
  - Blank 100 Number Grid (1 for each student)
  - 10-Stick (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup*."

Opening Exercise

2. Conduct the 30-second Opening Exercise you developed in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead several choral count-alouds by 10s from 0–100. Then lead a count-aloud in reverse from 100–0, which will necessarily be slower.

4. Then lead choral count-alouds by 1s—for short stretches of numbers—from various starting numbers that you call out. Focus on stretches of numbers that your students need to strengthen. For example, if they are wobbly going from the 90s to the 100s, count 95–105 several times rather than counting once from 75 to 125.

Round Robin

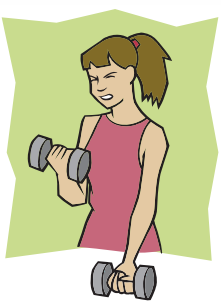
5. Conduct the Round Robin, counting by 10s from 0–300 and then in reverse from 300–0. Interrupt periodically with a HALT gesture, calling out re-directions, such as:
   - *Count backwards from there*
   - *Count forwards again*
   - *Start over at 130*
   - *Reverse the round-robin counting direction*

Number Grid

6. Students fill in the Missing Numbers 101–200 Grid. Coach any who need help connecting their counting skills to do this. Keep the number grid work to a minute or less.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout
(10–15 minutes)

Talk: + and -10

1. Explain that counting forward by 10 is the same as adding 10. Model adding and counting for students, then ask them to think of and share their own examples.

Variations:

Use more challenging examples (say 50–60, or 80–90, or 150–160), if appropriate for your particular students.

- *Let's count together by 10s from 0–100 (proceed to do so)*
- *So, now you know that 30 plus 10 is ___*
- *And 40 plus 10 is ___*
- *And 50 plus 10 is ___*

2. Explain that counting backwards by 10s is the same as subtracting 10. Model subtracting and counting for students, then ask them to think of and share their own examples.

Variations:

Use more challenging examples, if appropriate for your particular students.

- *Let's count together again by 10s backwards from 100–0 (proceed to do so)*
- *So, now you know that 100 minus 10 is ___*
- *And 90 minus 10 is ___*
- *And 80 minus 10 is ___*

3. Talk together about the different words for "+" (plus 10, 10 more, add 10, adding 10, addition) and the different words for "-" (minus 10, 10 less, take away 10, subtract 10, subtracting 10, subtraction).

4. Show and read cards from the + and - 10 Card Pack. On the Filled-in 100 Number Grid, explain and show how to move forward or back 10, depending upon the problem on the card.

5. Have students read +10 and -10 problems from the cards, answer, and show them on 100 Number Grid.

Explore: Using the 10-Stick

6. Give each student a 0–300 Number Line and a 10-Stick.

7. Lead your students in an exploration of their Number Lines and 10-Sticks, first +10 and then -10. Ask each student in turn to place his or her 10-Stick on a number you specify (a multiple of 10) and tell where it lands. Call out multiples of 10 in the range from 0–200 (for example, 30, 70, 90, 110, 130, 190).

Have students verbalize what the 10-Stick represents. Ask discussion questions such as:

- *What are you measuring with? [my 10-Stick]*
- *And when you start it on 30, where will it land? [on 40]*
- *So, why do we call it a 10-Stick? [because it measures 10 on the number line]*
- *And when you start at 40 and point your 10-Stick back (towards 0), where does the end of the 10-Stick land? [on 30]*

One-Minute Worksheet

8. Provide the One-Minute Worksheet for this lesson and time students for 60 seconds. Remind them that they can think about + and - 10 to solve problems and use what they've learned about counting by tens.

9. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 201–300. Add the completed grids to their Number Grid Folders.

Tip:

Coaching Students

Independent number grid work could provide important coaching time, so stay attentive to student needs and monitor progress. Teachers often leave their students unattended during this type of independent time, missing opportunities to clarify any confusion and correct the course of a student's thinking. On the other hand, take care not to interfere with students' quiet thinking. Attentiveness without interference is a balancing act.

To recognize a coaching opportunity:

- Look for students who are having trouble completing their grids. They may be working very slowly or not focusing on the task.
- Scan student work and look for errors. Take the opportunity to question the student in order to understand their thinking.

Reflect

2. Reflect on things that would be easy to count by 10s (e.g., dimes, number of fingers in a classroom, etc.).

3. Reflect with students about + and - 10 and about all the names of +10 and -10.

4. Reflect on filling in the number grid from 201 up.

- *What was hard?*
- *What strategies did you use?*

Paperclips

5. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

+10 and -10 Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

10+10=	200-10=	165-10=	256+10=	189-10=
100+10=	300-10=	95+10=	256-10=	214+10=
200+10=	15+10=	195+10=	38+10=	214-10=
300+10=	15-10=	295+10=	38-10=	104+10=
11+10=	25+10=	105-10=	97+10=	104-10=
101+10=	25-10=	205-10=	97-10=	291+10=
201+10=	125+10=	305-10=	207+10=	291-10=
301+10=	125-10=	137+10=	207-10=	199+10=
100-10=	165+10=	137-10=	189+10=	199-10=

FAST Math +−÷x

Name: _____ Date: _____

Missing Numbers 101-200 Grid

Fill in the missing numbers on the grid below.

101	102		104	105	106		108	109	110
	112	113		115	116	117		119	120
121		123	124	125		127	128	129	
131	132		134		136	137	138		140
141	142		144	145		147	148	149	
	152	153	154	155	156		158		160
	162	163	164		166	167		169	170
171	172		174	175		177	178		180
181	182		184	185	186		188	189	
	192	193	194		196	197		199	200

FAST Math +−÷x

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

10-Stick

Name: _____

Date: _____

				10-Stick					
				10-Stick					
				10-Stick					
				10-Stick					
				10-Stick					

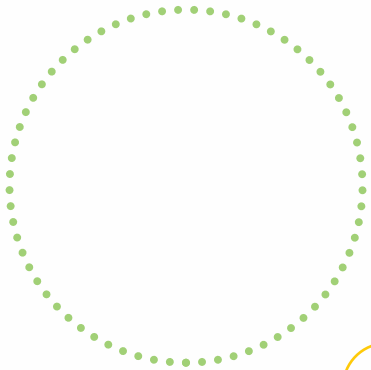
FAST Math +−÷x

Name: _____ Date: _____

Lesson 4: One-Minute Worksheet

10+10=	30+10=	40 -10	20 +10	20-10=	40+10=
50 +10	30 -10	60+10=	50-10=	80 -10	10 -10
70+10=	80-10=	110 +10	60 -10	70-10=	90-10=
150 +10	120 +10	140-10=	100+10=	100 -10	280 -10
140+10=	160-10=	190 +10	260 -10	250+10=	280+10=
130 +10	230 +10	260+10=	270-10=	210 +10	230 -10
240+10=	210-10=	290 -10	240 -10	230+10=	190-10=
150 -10	270 +10	170+10=	170-10=	300 -10	250 -10
90+10=	180-10=	200 +10	120 -10	130+10=	110-10=
180 +10	160 -10	200 -10	290 +10	300 +10	300 -10

FAST Math +−÷x



Lesson 5:

Counting by 5s

Learning objectives

- Count by 5s forward and in reverse (within the 5-count sequence)
- Establish the connection between counting up-and-down by 5 and adding and subtracting 5 (within the 5-count sequence)

Counting by 5s (multiples of 5) builds upon the skill of counting up-and-down the decades. Moving easily up or down the 5-scale is a mental math skill with much practical utility (e.g., nickels, clock face). When students can readily think "5 more" and "5 less," they have gained a valuable foundation skill.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather the materials specific to this lesson:
  - Completed number grid from students' collection
  - Number Squares 1–100 (from Lesson 2)
- Print the PDFs for Lesson 5:
  - Filled-in 100 Number Grid (with each multiple of 5 circled, 1 for the group)
  - +5 and -5 Card Pack (1 for the group)
  - Blank 100 Number Grid (1 for each student)
  - 5-Stick (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

Opening Exercise

2. Conduct the 30-second Opening Exercise you developed in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead several choral count-alouds by 10s from 0–300. Then lead a count-aloud in reverse from 300–0, which will necessarily be slower.
4. Lead several choral count-alouds by 5s from 0–300.
5. Then lead choral count-alouds by 5s—for short stretches of numbers—from various starting numbers that you call out.

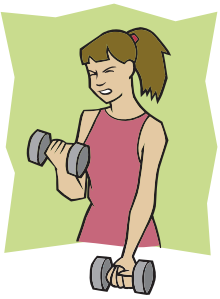
Round Robin

6. Conduct the Round Robin, counting by 5s. Interrupt the Round Robin with a HALT gesture, calling out re-directions, such as:
   - *Count backwards by 5s from there*
   - *Count forwards by 5s again*
   - *Reverse relay direction*

Number Grid

7. Using a Filled-in 100 Number Grid from their collection, have students draw squares around all the multiples of 5.
8. Then pose questions about the multiples they have outlined such as:
   - *How many 5s are in each row?*
   - *How many 5s are there between 1 and 100?*
9. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: + and - 5

1. Explain that counting forward by 5s is the same as adding 5. Model adding and counting for students, then ask them to think of and share their own examples.

Variations:

Use more challenging examples (say 55–60, or 80–95, or 155–160), if appropriate for your particular students.

- *Let's count together by 5s from 0–100*
- *So, now you know that 30 plus 5 is ___*
- *And 35 plus 5 is ___*
- *And 40 plus 5 is ___*

2. Explain that counting backwards by 5s is the same as subtracting 5. Model subtracting and counting for students, then ask them to think of and share their own examples.

Variations:

Use more challenging examples, if appropriate for your particular students.

- *Let's count together again by 5s from 100–0*
- *So, now you know that 100 minus 5 is ___*
- *And 95 minus 5 is ___*
- *And 85 minus 5 is ___*

3. Talk together about the different words for "+" (plus 5, 5 more, add 5, adding 5, addition) and the different words for "-" (minus 5, 5 less, take away 5, subtract 5, subtracting 5, subtraction).

4. Show and read cards from the +5 and -5 Card Pack. On the Filled-in 100 Number Grid, explain and show how to move forward or back 5, depending upon the problem on the card.

5. Have students read + and - 5 problems from the cards, answer, and show them on the Filled-in 100 Number Grid.

Explore: Using the 5-Stick

6. Give each student a 0–300 Number Line and a 5-Stick.

7. Lead your students in an exploration of their number lines and 5-Stick, first +5 and then -5. Ask each student in turn to place his or her 5-Stick on a number you specify (a multiple of 5) and tell where it lands. Call out multiples of 5 in the range from 0–200 (for example, 30, 55, 75, 100, 115, 195).

Have students verbalize what the 5-Stick represents. Ask discussion questions such as:

- *What are you measuring with? [my 5-Stick]*
- *And when you start it on 30, where will it land? [on 35]*
- *So, why do we call it a 5-Stick? [because it measures 5 on the number line]*

- *And when you start at 40 and point your 5-Stick back (towards 0), where does the end of the 5-Stick land? [on 35]*

One-Minute Worksheet

8. Provide the One-Minute Worksheet and time students for 60 seconds. Remind them that they can think about + and - 10 and + and - 5 to solve problems and use what they've learned about counting by 10s and by 5s.

9. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown
(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 301–400. Add the completed grids to their Number Grid Folders.

Reflect

2. Reflect on how easy it is to add and subtract 5 when you are really good at counting by 5s.

3. Discuss things that are easy to count by 5s (e.g., nickels, number of fingers in the classroom, 5-minute intervals on a clock face, etc.).

Paperclips

4. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

+5 and -5 Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

5+5=	50+5=	95+5=	40-5=	85-5=	125-5=	170-5=
10+5=	55+5=	100+5=	45-5=	90-5=	130-5=	175+5=
15+5=	60+5=	5-5=	50-5=	95-5=	135+5=	180-5=
20+5=	65+5=	10-5=	55-5=	100-5=	140-5=	185+5=
25+5=	70+5=	15-5=	60-5=	105-5=	145+5=	190+5=
30+5=	75+5=	20-5=	65-5=	105+5=	150+5=	195-5=
35+5=	80+5=	25-5=	70-5=	110+5=	155-5=	195+5=
40+5=	85+5=	30-5=	75-5=	115-5=	160+5=	200-5=
45+5=	90+5=	35-5=	80-5=	120+5=	165-5=	200+5=

FAST Math +−÷x

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

5-Stick

Name: _____

Date: _____

5-Stick	5-Stick
5-Stick	5-Stick
5-Stick	5-Stick
5-Stick	5-Stick
5-Stick	5-Stick

FAST Math +−÷×

Name: _____ Date: _____

Lesson 5: One-Minute Worksheet

15+5=	20+10=	50 +10	5 +5	10-5=	10+10=
60 -10	30 -5	25-5=	20+5=	70 +10	25 +5
40+5=	45-5=	50 +5	30 -10	55-5=	90+10=
100 -10	90 +5	35-5=	85+5=	60 -5	110 +10
100-5=	115-5=	40 -10	80 -10	135+5=	65+5=
195 -5	110 +5	165-5=	140-10=	145 +5	150 -5
95-5=	195+5=	180 +5	130 -10	105-5=	180+10=
200 -10	75 +5	200-5=	185+5=	160 +10	105 -5
140+5=	190+10=	170 -5	170 -10	160+5=	80-5=
150 +10	175 -5	155+5=	120-10=	120 +5	125 -5

FAST Math +−÷x

Lesson 6:

Counting On and Back by 10s from Any Number

Learning objectives

- Practice counting by 10, both forward and reverse, from any number
- Reinforce the connection between counting by 10 and addition/subtraction

Increasing and decreasing by 10, not only at the decades but from any number, demonstrates a robust understanding of our number system. As math-thinking skills increase and become well-practiced, they are like gymnastic moves on the monkey bars of the mind. Students have developed a powerful repertoire when they can nimbly count far, count on, count the decades, play up and down the 5 scale, and on top of that, can add and subtract 10 from any number. These counting skills build a foundation for more advanced counting strategies.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather the materials specific to this lesson:
  - Number Squares 1–100 (from Lesson 2)
- Print the PDFs for Lesson 6:
  - +10 and -10 Card Pack (1 for each student)
  - Blank 100 Number Grid (1 for each student)
  - Filled-in 100 Number Grid (1 for each student)
  - Missing Numbers 101–200 Grid (from Lesson 4, 1 for each student)
  - 10-Stick (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead the class in three count-alouds. Call out a number (0–300) and ask students to count by 10. Choose a different starting number and repeat the process.

4. Lead the class in three backwards count-alouds. Call out a number (0–300) and ask students to count backwards by 10. Choose a different starting number and repeat the process.

Round Robin

5. Conduct the Round Robin, counting by 10 and starting at a random number. Interrupt the Round Robin with a HALT gesture, calling out re-directions such as:

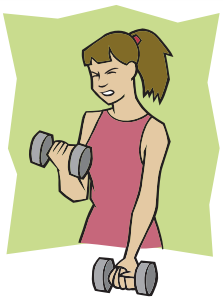
- *Count backwards*
- *Restart at 23 and count forward by 10*
- *Reverse round robin counting direction*

Number Grid

6. Give students the Missing Numbers 101–200 Grid and have them fill in missing numbers.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: + or -10 from Any Number

1. Discuss the similarities between counting forward by 10 and addition (adding 10 to a number). Model adding and counting for students, then ask them to think of and share their own examples (e.g., 65, 75, 85, 95, etc.).

2. Explain that counting backwards by 10 is the same as subtracting 10. Model subtracting and counting for students, then ask them to think of and share their own examples (e.g., 145, 135, 125, 115, 105, etc.).

3. Talk again about words for "+" (plus, more, add, adding, addition) and for "-" (minus, less, take away, subtract, subtracting, subtraction).

4. Show and read cards from the +10 and -10 Card Pack. On the Filled-in 100 Number Grid, explain and show how you move forward or backward 10, depending upon the problem on the card.

5. Have students read + or - 10 problems from the cards, answer, and show them on the Filled-in 100 Number Grid using the different "+" and "-" vocabulary words.

Explore: + or -10 Neighbors

6. Demonstrate several examples of finding numbers that are -10 and +10 using your Filled-in 100 Number Grid. On their blank grids, have students pull the following number squares and place them on the correct spots (11, 22, 33, 44, 55, 66, 77, 88). Then have students determine and place on their grids those number squares that are the +10 and -10 neighbors of those numbers.

One-Minute Worksheet

7. Provide the One-Minute Worksheet and time students for 60 seconds. Remind them that they can think about + or - 10 and + or - 5 to solve problems.

8. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown
(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 401–500. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on how easy it is to add and subtract by 10 from any number.

3. Draw links to "problem-solving" applications by asking discussion questions such as:

- *So let's say you had 14 cents. If someone gave you another dime, how much money would you have? [24 cents]*
- *Then you found two more dimes. How much would you have? [44 cents]*
- *What if you spent three of your dimes on gum? [14 cents again]*

Paperclips

4. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

+10 and -10 Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

10+10=	200-10=	165-10=	256+10=	189-10=
100+10=	300-10=	95+10=	256-10=	214+10=
200+10=	15+10=	195+10=	38+10=	214-10=
300+10=	15-10=	295+10=	38-10=	104+10=
11+10=	25+10=	105-10=	97+10=	104-10=
101+10=	25-10=	205-10=	97-10=	291+10=
201+10=	125+10=	305-10=	207+10=	291-10=
301+10=	125-10=	137+10=	207-10=	199+10=
100-10=	165+10=	137-10=	189+10=	199-10=

FAST Math +−÷x

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Missing Numbers 101-200 Grid

Fill in the missing numbers on the grid below.

101	102		104	105	106		108	109	110
	112	113		115	116	117		119	120
121		123	124	125		127	128	129	
131	132		134		136	137	138		140
141	142		144	145		147	148	149	
	152	153	154	155	156		158		160
	162	163	164		166	167		169	170
171	172		174	175		177	178		180
181	182		184	185	186		188	189	
	192	193	194		196	197		199	200

FAST Math +−÷x

10-Stick

Name: _____

Date: _____

				10-Stick					
				10-Stick					
				10-Stick					
				10-Stick					
				10-Stick					

FAST Math +−÷x

Name: _____ Date: _____

Lesson 6: One-Minute Worksheet, Part A

10+10=	30+10=	20 -10	40 -10	70-10=	70+10=
50 -10	40 +10	90-10=	80+10=	60 +10	100 -10
15+10=	90+10=	25 -10	35 +10	15-10=	25+10=
55 -10	55 +10	85+10=	85-10=	95 -10	95 +10
3+10=	11+10=	17 -10	17 +10	21+10=	21-10=
34 +10	37 -10	39-10=	41+10=	56 -10	52 +10
19-10=	61+10=	91 +10	91 -10	77+10=	63-10=
13 -10	99 +10	101-10=	101+10=	97 +10	98 -10
103-10=	103+10=	99 -10	89 -10	99+10=	1+10=
105 -10	106 +10	96-10=	93+10=	111 -10	101 +10

FAST Math +−÷x

Name: _____ Date: _____

Lesson 6: One-Minute Worksheet, Part B

1+10=	6+10=	50 +10	51 +10	24-10=	65-10=
31 -10	32 +10	38-10=	6+10=	99 -10	91 +10
61-10=	59-10=	47 -10	74 +10	11-10=	120+10=
133 -10	200 +10	111-10=	63-10=	55 +10	173 -10
49 +10	111 -10	198+10=	211-10=	200 +10	200 -10
201+10=	201-10=	53 +10	159 -10	88+10=	78-10=
29 +10	100 -10	95+10=	183-10=	73 +10	66 -10
103 +10	107 -10	105-10=	105+10=	89 +10	219 +10
134+10=	133-10=	172 +10	201 -10	77+10=	77-10=
83 +10	91 -10	191+10=	191-10=	102 -10	197 +10

FAST Math +−÷x

Lesson 7:

Counting by 2s (Even Numbers)

Learning objectives

- Develop students' ability to count by 2s from any even number
- Identify numbers in the even-number count sequence as even numbers

It is important to practice counting by 2s, or skip-counting, until it flows easily. Sometimes this is not as easily developed as counting by 10s or 5s, since at first it requires students to actively suppress the automatized counting-by-1s.

It is also essential for students to identify numbers as "even" before proceeding to the odd number count sequence. Otherwise, the two sequences and the even/odd labels often become confused and extremely hard to untangle.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 7:
  - Filled-in 100 Number Grid (1 for each student)
  - Even Missing Numbers 1–100 Grid (1 for each student)
  - Blank 100 Number Grid (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup*."

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students counting by 2s from 0–50.

4. Lead counting-on by 2s for short stretches from any even number you call out from 1–100.

Round Robin

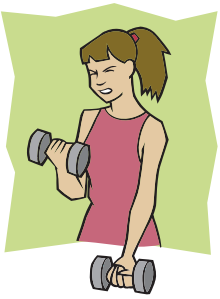
5. Conduct the Round Robin, counting by 2s (even numbers). Interrupt periodically with a HALT gesture to reverse direction or start at a different even number.

Number Grid

6. Ask students to fill in missing even numbers on the Even Missing Numbers 1–100 Grid.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: Counting by 2s (even numbers)

1. Clarify that when you count by 2s, starting with 2, all the numbers are called even numbers. Ask the following questions so that the students use the term "even" or "even number" when talking about numbers:

- *What kind of numbers are you counting? [even]*
- *What kind of number was the last number you counted? [even]*

2. Explain that counting forward by 2s is like counting by 1s, but skipping every other number (that's why we also call it skip-counting). Demonstrate and ask students to demonstrate skip-counting aloud.

3. Explain that counting forward by 2s is also the same as adding two. Model adding and counting for students, then ask them to think of and share their own examples.

- *Let's count together again by 2s from 50–60 (proceed to do so)*
- *So, now you know that 50 plus 2 is ___*
- *And 52 plus 2 is ___*
- *And 54 plus 2 is ___*

4. Explain that counting backwards by 2s is the same as subtracting. Model subtracting and counting for students, then ask them to think of and share their own examples.
   - *Let's count together again by 2s from 60–50 (proceed to do so)*
   - *So, now you know that 60 minus 2 is ___*
   - *And 58 minus 2 is ___*
   - *And 56 minus 2 is ___*

Explore: Counting by 2s (even numbers)

5. Have students count by 2s on their Filled-in 100 Number Grid. As they count each number, they cross out numbers they are not saying (the odd numbers). By the time they reach 100, they now have a grid with only even numbers on it. Have them describe the patterns they see. Stress that all of the numbers showing are even numbers.
   - *How many even numbers are there?*
   - *What numbers repeat?*
   - *What is the highest even number on your grid?*

One-Minute Worksheet

6. Provide the One-Minute Worksheet and time students for 60 seconds. Remind them that they can think about + and - 10, + and - 5, and counting by 2s to solve problems.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 501–600. Add the completed grids to the collection started in Lesson 1.

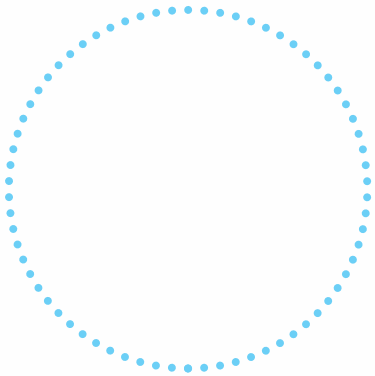
Reflect

2. Reflect together on how easy it is to add and subtract by 2s when you are really good at counting by 2s.

3. Reflect on things that would be easy to count by 2s (e.g., shoes, socks, students lined up in pairs, etc.).

Paperclips

4. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.



Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Even Missing Numbers 1-100 Grid

Fill in the missing numbers on the grid below.

1		3		5		7		9	
11		13		15		17		19	
21		23		25		27		29	
31		33		35		37		39	
41		43		45		47		49	
51		53		55		57		59	
61		63		65		67		69	
71		73		75		77		79	
81		83		85		87		89	
91		93		95		97		99	

FAST Math +−÷x

Name: _____ Date: _____

Lesson 7: One-Minute Worksheet

2+2=	25+5=	30 +10	4 -2	6+2=	90+10=
50 -5	50 -10	10-2=	12+2=	100 -5	16 +2
18-2=	18+2=	100 -10	20 -2	20+2=	65+5=
32 -2	32 +2	28+2=	95+5=	95 -10	30 -2
11+10=	11-10=	50 -2	50 +2	46-2=	44+2=
70 +5	70 -10	70+2=	70-2=	76 +2	78 -2
90+2=	90+5=	90 -5	90 -2	78+2=	80-5=
80 -2	46 +2	52+2=	52+10=	37 -10	36 -2
88 +2	100 -2	96-2=	94-2=	98 +2	88 +10
91+10=	98-2=	100 +2	104 -2	102+2=	100-2=

FAST Math +−÷x

Lesson 8:

Counting by 2s (Odd Numbers)

Learning objectives

- Count by 2s from any odd number
- Identify numbers in the odd-number count sequence as odd numbers

Skip-counting from odd numbers takes more concentration than counting from even numbers, so we do not expect it to be quite as smooth. It is sufficient for students to be slow with this as long as they are accurate. Proceed to the next lesson once students can do it and can identify numbers as "odd."

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather the materials specific to this lesson:
  - Deck of cards with face cards (royals) removed for Odd/Even Snap
- Print the PDFs for Lesson 8:
  - Filled-in 100 Number Grid (1 for the group)
  - Odd Missing Numbers 1–100 Grid (1 for each student)
  - Blank 100 Number Grid (1 for each student)
  - One-Minute Worksheet (1 for each student)
  - Odd/Even Snap Game Card
  - Odd/Even Finger Flash Game Card (optional)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by 2s from 1–51.

4. Then lead counting-on by 2s for short stretches from any odd number you call out from 1–99.

Round Robin

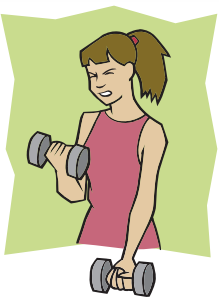
5. Conduct the Round Robin counting by 2s (odd). Interrupt periodically with a HALT gesture to reverse direction or start at a different odd number.

Number Grid

6. Using an Odd Missing Numbers 1–100 Grid, students fill in missing (odd) numbers.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

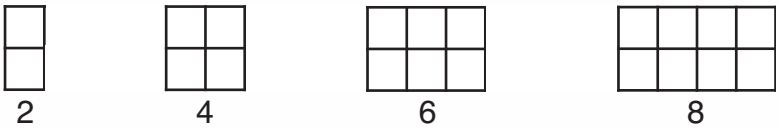
(10–15 minutes)

Talk: Counting by 2s (odd numbers)

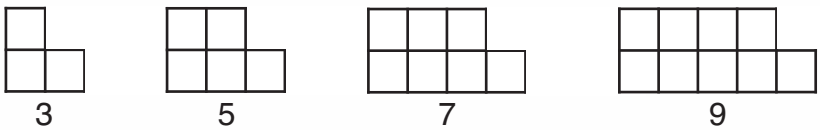
1. Clarify that when you count by 2s, starting with 1, all the numbers are called odd numbers. Ask the following questions so that the students use the term "odd" or "odd number" when talking about numbers:

- *What kind of numbers are you counting? [odd]*
- *What kind of number was the last number you counted? [odd]*

2. Demonstrate by drawing how even numbers are even:



And how odd numbers stick out; one doesn't have a partner:



Explore: Counting by 2s (odd numbers)

3. Play Odd/Even Snap, according to the directions on the game card.

Odd/Even Snap

Number of Players: 2

Materials:
- **Deck of cards with royals (kings, queens, jacks) removed (any card deck with cards numbered 1-10 can be used)**

Divide the cards evenly between the two players. Cards are kept face down in a stack in front of each player. For each round, both players show the top card on his/her stack. Players have to look at the two cards facing up and say "both even", "both odd", or "mixed". The first player to call correctly gets the cards. In the case of a tie, each player keeps his/her card. Play ends when players have gone through their stacks once. The player with the most cards wins.

Note: If using a regular deck of cards, teach that the ace is the same as 1 and is odd.

One-Minute Worksheet

4. Provide students the One-Minute Worksheet and time them for 60 seconds. Remind them that they can think about + and - 10, + and - 5, and counting by 2s to solve problems.

5. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

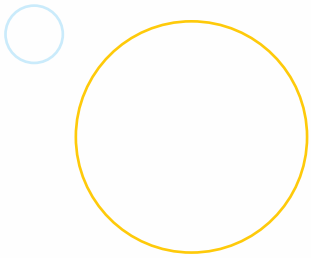
1. Have each student fill in a Blank Number Grid from 601–700. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on how skip-counting odd numbers is different from skip-counting even numbers and on which one is more difficult.

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.



Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Odd Missing Numbers 1-100 Grid

Fill in the missing numbers on the grid below.

	2		4		6		8		10
	12		14		16		18		20
	22		24		26		28		30
	32		34		36		38		40
	42		44		46		48		50
	52		54		56		58		60
	62		64		66		68		70
	72		74		76		78		80
	82		84		86		88		90
	92		94		96		98		100

FAST Math +−÷×

Name: _____ Date: _____

Lesson 8: One-Minute Worksheet

80 +10	25 -5	3-2=	18-2=	5 -2	5 +2
28+2=	17-2=	19 +2	18 +2	100-10=	100-5=
15 +2	15 -2	95+5=	92-2=	92 -10	13 -2
31-2=	80-5=	29 +2	29 +10	29-10=	29-2=
41 +2	41 -2	41+10=	41-10=	100 -5	100 -2
55+2=	55-2=	59 +2	51 -2	69+10=	69+2=
71 +2	73 -2	71-2=	70-2=	75 +2	79 +2
98+2=	90-5=	90 -2	89 +2	89-2=	77-2=
83-2=	83-10=	98 -2	79 +2	100+5=	59+2=
105-5=	105-10=	98 +2	99 -2	91-2=	79+2=

FAST Math +−÷x

Name: _____ Date: _____

Odd/Even Snap Game Card

Follow the instructions on the game card below to play Odd/Even Snap.

Odd/Even Snap

Number of Players: 2

Materials:

- **Deck of cards with royals (kings, queens, jacks) removed (any card deck with cards numbered 1-10 can be used)**

Divide the cards evenly between the two players. Cards are kept face down in a stack in front of each player. For each round, both players show the top card on his/her stack. Players have to look at the two cards facing up and say "both even", "both odd", or "mixed". The first player to call correctly gets the cards. In the case of a tie, each player keeps his/her card. Play ends when players have gone through their stacks once. The player with the most cards wins.

Note: If using a regular deck of cards, teach that the ace is the same as 1 and is odd.

FAST Math +−÷x

Name: _____ Date: _____

Odd/Even Finger Flash Game Card

Follow the instructions on the game card below to play Odd/Even Finger Flash.

Odd/Even Finger Flash

Number of Players: **2-4**

Materials:
- **2 chips or tiles**
- **Piece of paper**
- **Pencil**

The first player makes a guess and places either 1 or 2 chips (representing odd or even) under a piece of paper without letting the other players see the amount.

All players shake their fists and then, on the count of "three," extend their hands with none, some, or all of their fingers showing. The first player must quickly count up the fingers and determine whether the total is "even" or "odd." The chips under the paper are then revealed.

Players can earn 0, 1, or 2 points in their round. Correctly identifying the total number of fingers as even or odd earns the player 1 point. If the player was incorrect, 0 points are earned. If the player was correct AND guessed the corresponding number of chips as even or odd, 2 points are earned.

For example:
- **Player One guesses the sum for the round will be odd**
- **Player One places 1 chip under the paper, representing his guess of "odd"**
- **Players extend a total of 11 fingers**
- **Player One says "odd"**
- **Player One earns two points (1 for guessing correctly, 1 for counting)**

Player One should record his/her point total for the round. Play continues until one player wins by earning 10 points.

FAST Math +−÷x

Lesson 9:

Counting by 2s (Even and Odd)

Learning objectives

- Count forward and back by 2 from any even or odd number
- Make the connection to adding and subtracting 2, focusing on the oddness or evenness of the resulting answer

The fact that skip-counting backwards is challenging can help solidify easier skills on the mental number line. Neither odd-number nor backwards skip-counting will ever be as nimble and effortless as counting forwards by 2s from even points (or counting by 1s, 10s, or 5s). It is okay if students remain slow. Still, exercising these mental moves can strengthen students' number sense as well as their powers of concentration.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 9:
  - Blank 100 Number Grid (2 for each student)
  - +2 and -2 Even Card Pack (1 for each student)
  - +2 and -2 Odd Card Pack (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by 2s (even) from 0–50. Remind them, "*OK, even number counting!*"

4. Then lead counting-on by 2s for short stretches from any even number you call out from 1–100.

5. Lead students in counting by 2s (odd) from 1 to 51. Remind them, *"Now, odd number counting!"*

6. Then lead counting-on by 2s for short stretches from any odd number you call out from 1–99.

Round Robin

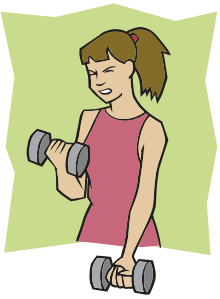
7. Conduct the Round Robin counting by 2s, switching from even to odd sequences from different starting numbers. Interrupt the Round Robin periodically with a HALT gesture to call out "ODD" and a new starting number, then "EVEN" and another starting number.

Number Grid

8. Using a Blank 100 Number Grid, students fill in all odd numbers and then all even numbers.

9. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: Adding and Subtracting 2s

1. Remind students that counting by 2s is the same as adding 2, and counting backwards by 2 is the same as subtracting 2. Clarify if needed.

Show a card from the +2 and -2 Even Card Pack and read the number sentence. For example: 4+2 =

- *The starting number is 4. What kind of number is that? [even]*
- *What's 4+2?*
- *Yes, 6. What kind of number is that? [even]*
- *So, what kind of a number do you get when you add an even number, plus 2?*

Show a card from +2 and -2 Odd Card Pack and read the number sentence. For example: 5+2 =

- *The starting number is 5. What kind of number is that? [odd]*
- *What's 5+2?*
- *Yes, 7. What kind of number is that? [odd]*
- *So, what kind of a number do you get when you add an odd number, plus 2?*

Provide one or two more examples, if needed.

Explore: + or - 2

2. Mix the two card packs together. Have a student turn over a card, say the number on it and label it "even" or "odd." Have another student turn over the mixed +2 and -2 pack and read it. The first student gives the "answer" and labels it "even" or "odd."

One-Minute Worksheet

3. Provide the One-Minute Worksheet and time students for 60 seconds. Remind them that they can think about + or - 2 to solve the problems.

4. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 701-800. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on how easy it is to add and subtract by 2s when you are really good at counting by 2s with both even and odd numbers.

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

+2 and -2 Even Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

10+2=	**60+2=**	**10-2=**	**12-2=**	**66-2=**
14+2=	**66+2=**	**20-2=**	**16-2=**	**78-2=**
18+2=	**72+2=**	**30-2=**	**24-2=**	**84-2=**
22+2=	**78+2=**	**40-2=**	**28-2=**	**86-2=**
26+2=	**80+2=**	**50-2=**	**32-2=**	**92-2=**
34+2=	**84+2=**	**60-2=**	**36-2=**	**94-2=**
38+2=	**88+2=**	**70-2=**	**44-2=**	**96-2=**
44+2=	**94+2=**	**80-2=**	**48-2=**	**98-2=**
52+2=	**96+2=**	**90-2=**	**56-2=**	
58+2=	**98+2=**	**100-2=**	**62-2=**	

FAST Math +−÷x

+2 and -2 Odd Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

9+2=	61+2=	11-2=	15-2=	69-2=
13+2=	67+2=	21-2=	19-2=	77-2=
17+2=	73+2=	31-2=	23-2=	83-2=
21+2=	79+2=	41-2=	27-2=	85-2=
27+2=	81+2=	51-2=	35-2=	93-2=
35+2=	85+2=	61-2=	39-2=	95-2=
39+2=	89+2=	71-2=	47-2=	97-2=
45+2=	95+2=	81-2=	49-2=	99-2=
53+2=	97+2=	91-2=	55-2=	
59+2=	99+2=	101-2=	63-2=	

FAST Math +−÷x

Name: _____ Date: _____

Lesson 9: One-Minute Worksheet

80 +10	25 -5	3-2=	18-2=	5 -2	5 +2
28+2=	17-2=	19 +2	18 +2	100-10=	100-5=
15 +2	15 -2	95+5=	92-2=	92 -10	13 -2
31-2=	80-5=	29 +2	29 +10	29-10=	29-2=
41 +2	41 -2	41+10=	41-10=	100 -5	100 -2
55+2=	55-2=	59 +2	51 -2	69+10=	69+2=
71 +2	73 -2	71-2=	70-2=	75 +2	79 +2
98+2=	90-5=	90 -2	89 +2	89-2=	77-2=
83-2=	83-10=	98 -2	79 +2	100+5=	59+2=
105-5=	105-10=	98 +2	99 -2	91-2=	79+2=

FAST Math +−÷x

Lesson 10:

Counting by 100s

Learning objectives

- Count by 100s forward and in reverse to 1,000
- Read and write numerals to 2,000
- Add and subtract 100 from any number from 100–1,000

Counting up to really big numbers is a challenge that can promote insight into the whole number system. Patterns students catch on to when counting by ones (…21, 22, 23…31, 32, 33…41, 42, 43) get extended when counting by tens (…42, 52, 62…), and then by hundreds (...453, 553, 653...). But the patterns may not begin to emerge until students are counting by thousands. Then it begins to dawn on them that this is a progressive—in a sense, endless—pattern. This higher counting can engender insight and deeper intuitive understanding of the system.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 10:
  - +100 and -100 Card Pack
  - Blank 100 Number Grid (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by 100s from 100–2,000.
4. Lead students in counting by 100s from 5–905.

5. Provide other starting points until they get the hang of it: 22–922, 35–935, 41–941, etc.

Round Robin

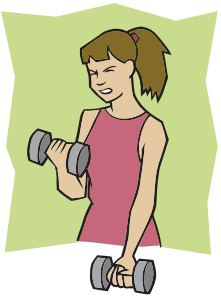
6. Conduct the Round Robin, counting by 100s. Interrupt periodically with a HALT gesture to reverse direction or start at a different number.

Number Grid

This lesson does not contain number grid work.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: Adding and Subtracting 100s

1. Explain to students that counting forward by 100s is the same as adding 100 to a number, and counting in reverse by 100 is the same as subtracting 100.

   Go through examples linking counting (100, 200, 300, 400, 500) to +100 problems: 100+100=200, 200+100=300, 300+100=400, until the students answer +100 questions with understanding.

   Remember, even though these are large numbers, you want to help students hear the counting logic. It's important to discuss and add verbally—do not write numerals to explain.

2. Explain that they can add 100 to any number. Give several examples, so they can catch the pattern: 25+100=125, 63+100=163, 99+100=199.

3. Explain that they can subtract 100 from any number. Give examples, so they can catch the pattern: 199-100=99, 163-100=63, 125-100=25.

Explore: + or - 100

4. Explore the quantity represented by numbers 100 and higher. First, have students line up and connect their individual 0–300 Number Lines to see how many hundreds that makes and to feel how long that many 100s are.

5. Then have students show differing +100 and -100 combinations using their number lines (e.g., 125+100/125-100, 150+100/150-100, 193+100/193-100).

One-Minute Worksheet

6. Provide the One-Minute Worksheet and time students for 60 seconds. Remind them that they can think about + or - 100 to solve the problems.

7. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown
(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 801–900. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on how easy it is to "think" the answer to +100 and -100 with any number. Reflect on any thinking strategies they used.

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

+100 and -100 Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

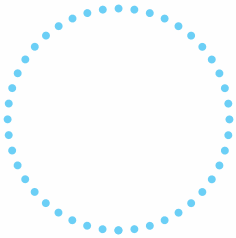
100+100=	105+100=	170-100=	195-100==	122-100=	207+100=
200+100=	135+100=	190-100=	205-100=	137+100=	207-100=
300+100=	165+100=	210-100=	215-100=	137-100=	214+100=
400+100=	195+100=	230-100=	255-100=	158+100=	214-100=
110+100=	215+100=	250-100=	285-100=	158-100=	231+100=
130+100=	255+100=	270-100=	305-100=	171-100=	231-100=
150+100=	275+100=	310-100=	104+100=	171+100=	256+100=
160+100=	295+100=	105-100=	104-100=	189+100=	256-100=
240+100=	200-100=	135-100=	111+100=	189-100=	289+100=
270+100=	300-100=	155-100=	119-100=	199+100=	289-100=
290+100=	400-100=	165-100=	122+100=	199-100=	291+100=
291-100=	297+100=	297-100=			

Name: _____ Date: _____

Lesson 10: One-Minute Worksheet

100+100=	150+100=	200 +100	240 +100	300-100=	170-100=
105 +100	210 -100	350-100=	230-100=	300 +100	270 -100
310 -100	165 +100	400-100=	195+100=	205 -100	295 +100
215-100=	104+100=	104 -100	111 +100	119-100=	165-100=
275 +100	137 -100	137+100=	285-100=	214 -100	214 +100
105-100=	158+100=	155 -100	207 +100	122-100=	122+100=
231+100=	207-100=	305 -100	158 -100	256+100=	189+100=
290 +100	255 +100	199-100=	199+100=	231 -100	270 +100
171-100=	171+100=	256 -100	291 +100	135-100=	289+100=
289 -100	255 -100	291-100=	189-100=	297 -100	297 +100

FAST Math +−÷x



Lesson 11:

Counting by 25s

Learning objectives

- Count by 25s forward and in reverse (within the 25 count sequence)
- Make the connection to adding and subtracting 25 (within the 25 count sequence)

Twenty-fives are valuable building blocks in our culture and easy to grasp. Becoming nimble at thinking + and - 25 (with multiples of 25) is handy. Furthermore, such mental math skills often promote feelings of genuine accomplishment, invigorating children's efforts and risk-taking.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather the materials specific for this lesson:
  - Paper bag and fist full of real or pretend quarters (for each pair of students)
- Print the PDFs for Lesson 11:
  - Blank 100 Number Grid (1 for each student)
  - x25 Missing Numbers 1–100 Grid (1 for each student)
  - +25 and -25 Card Pack (1 for each student)
  - Fist Full of Coins Game Card (1 for each student pair)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup.*"

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by 25s from 0–525.

4. Lead students in counting by 25s from multiples of 25 that you call out.

Round Robin

5. Conduct the Round Robin, counting by 25s from 0–500 and in reverse from 500–0.

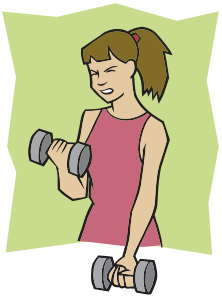
6. Call out different starting numbers that are multiples of 25 as the counting goes around. Interrupt the Round Robin periodically with a HALT gesture and have the counting proceed in reverse.

Number Grid

7. Using a x25 Missing Numbers 1–100 Grid, students fill in missing multiples of 25.

8. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout
(10–15 minutes)

Talk: Adding and Subtracting 25

1. Choose a card from the +25 and -25 Card Pack. Read the number sentence aloud and explain how you arrive at your answer. Then, display another card from the +25 and -25 Card Pack and have students read and solve the problem with you.

Explore: Fist Full of Quarters

2. Group students into pairs.

3. Follow the instructions on the Fist Full of Coins Game, using only quarters.

Fist Full of Coins Game

Number of Players: 2

Materials:
- coins in a bag

Each player reaches into the paper bag (without peeking) and grabs a small fistful of coins. Players count the value of their coins. The player with the higher value wins the round and keeps his/her coins. The other player returns his/her coins to the bag. If a player catches an opponent miscounting, that player wins the round. Play until the bag is empty. The player with the most coins at the end of the game wins.

One-Minute Worksheet

4. Provide the One-Minute Worksheet and time students for 60 seconds. Remind them that they can think about + or - 25 to solve the problems.

5. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown
(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 901–1000. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on counting by 25s.

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

x25 Missing Numbers 1-100 Grid

Fill in the missing numbers on the grid below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24		26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74		76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	

FAST Math +−÷x

+25 and -25 Card Pack

Copy these number sentences by hand onto 3" x 5" index cards or print them out onto labels. Select the examples most appropriate for your students.

25+25=	**250+25=**	**475+25=**	**200-25=**	**425-25=**
50+25=	**275+25=**	**500+25=**	**225-25=**	**450-25=**
75+25=	**300+25=**	**25-25=**	**250-25=**	**475-25=**
100+25=	**325+25=**	**50-25=**	**275-25=**	**500-25=**
125+25=	**350+25=**	**72-25=**	**300-25=**	
150+25=	**375+25=**	**100-25=**	**325-25=**	
175+25=	**400+25=**	**125-25=**	**350-25=**	
200+25=	**425+25=**	**150-25=**	**375-25=**	
225+25=	**450+25=**	**175-25=**	**400-25=**	

FAST Math +−÷x

Name: _____ Date: _____

Fist Full of Coins Game Card

Variations:
The game can be played with one type of coin as well as mixed coins. Initially, have the students play with one type of coin (nickels, dimes, or quarters) as appropriate to the lesson. Starting with Lesson 12, mix the coins together.

Fist Full of Coins

Number of Players: 2

Materials:
- **coins in a bag**

Each player reaches into the paper bag (without peeking) and grabs a small fistful of coins. Players count the value of their coins. The player with the higher value wins the round and keeps his/her coins. The other player returns his/her coins to the bag. If a player catches an opponent miscounting, that player wins the round. Play until the bag is empty. The player with the most coins at the end of the game wins.

FAST Math +−÷x

Name: _____ Date: _____

Lesson 11: One-Minute Worksheet

25+25=	100+25=	50 +25	100 -25	50-25=	125-25=
75 -25	75 +25	125+25=	250+25=	150 -25	225 +25
175-25=	200+25=	250 -25	150 +25	175+25=	225-25=
275 +25	200 -25	25-25=	300+25=	350 -25	100 -25
300-25=	400-25=	325 +25	175 +25	375-25=	475+25=
325 +25	275 -25	450+25=	375-25=	450 -25	350 +25
425-25=	450+25=	275 +25	475 -25	300-25=	375+25=
25 -25	75 +25	225-25=	425+25=	350 -25	350 +25
275+25=	300-25=	475 -25	475 +25	375 -25	250 +25
400 +25	200 -25	375+25=	500-25=	400 -25	475 +25

FAST Math +−÷x

Lesson 12:

Counting by 1s, 5s, 10s, 25s, and 100s

Learning objectives

- Count by 1s, 5s, 10s, 25s, and 100s in various combinations and patterns
- Connect counting strategies with adding and subtracting coin values (Knowledge of coin values 5¢, 10¢, 25¢, and 100¢=$1 is assumed. Take the time to teach these, if needed.)

Students will be switching rapidly from one counting sequence to another. They are demonstrating that they can count nimbly and flexibly, which is essential in developing foundational counting skills.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather materials specific to this lesson:
  - Paper bag and fists full of real or pretend pennies, nickels, dimes, and quarters (for each pair of students)
- Print the PDFs for Lesson 12:
  - Blank 100 Number Grid (1 for each student)
  - Fist Full of Coins Game Card
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup*."

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by 1s, 5s, 10s, 25s, and 100s in a mixed sequence. For example, count:

- *by 2s (odd) from 1 to 35*
- *then switch—by 5s from 35–100*
- *then switch—by 10s from 100–150*
- *then switch—by 25s from 150–350*
- *then switch—by 100s from 350–1,050*

If these sequences are difficult, practice several times with the same sequences or similar ones.

Round Robin

4. Conduct the Round Robin, counting:

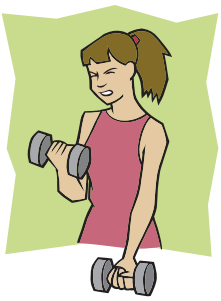
- *by 2s (odd) to 35*
- *by 5s to 100*
- *by 10s to 150*
- *by 25s to 350*
- *by 100s to 1,050*

Number Grid

This lesson does not contain number grid work.

5. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: Counting Money

1. Model for students how to count up money. Start with the $1 bills and count by 100 (cents), then count the quarters by counting-on by 25 cents. Continue by counting-on dimes, nickels, and pennies.

2. Pose money amounts and have the students direct you in counting out that amount.

Explore: Fist Full of Coins

3. Put students into pairs. Follow the instructions on the game card to play the Fist Full of Coins Game.

Fist Full of Coins Game

Number of Players: 2

Materials:
• coins in a bag

Each player reaches into the paper bag (without peeking) and grabs a small fistful of coins. Players count the value of their coins. The player with the higher value wins the round and keeps his/her coins. The other player returns his/her coins to the bag. If a player catches an opponent miscounting, that player wins the round. Play until the bag is empty. The player with the most coins at the end of the game wins.

One-Minute Worksheet

4. Provide the One-Minute Worksheet and time students for 60 seconds. Remind students that they can think about what they know to solve the problems.

5. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown
(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 1,001–1,100. Be sure to help them do this correctly. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on counting money.
   - *What's easy?*
   - *What's not so easy?*

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Fist Full of Coins Game Card

Variations:
The game can be played with one type of coin as well as mixed coins. Initially, have the students play with one type of coin (nickels, dimes, or quarters) as appropriate to the lesson. Starting with Lesson 12, mix the coins together.

Fist Full of Coins

Number of Players: 2

Materials:
- **coins in a bag**

Each player reaches into the paper bag (without peeking) and grabs a small fistful of coins. Players count the value of their coins. The player with the higher value wins the round and keeps his/her coins. The other player returns his/her coins to the bag. If a player catches an opponent miscounting, that player wins the round. Play until the bag is empty. The player with the most coins at the end of the game wins.

FAST Math +−÷x

Name: _____ Date: _____

Lesson 12: One-Minute Worksheet

45-5=	53+10=	100 -2	137 +10	200-25=	99+2=
90 +10	111 -10	125-5=	65+5=	135 -100	265 +100
19-2=	98-2=	200 -25	375 +25	71-2=	71+2=
207 +100	205 -5	190+10=	189+100=	185 -5	89 +2
300-25=	300-5=	300 -100	300 -10	100-2=	71+2=
95 +5	80 -5	101+10=	101+10=	101 -10	100 -25
107-10=	107+10=	99 -2	19 +2	297+100=	256-100=
171 +100	93 +2	93+10=	97+100=	256 -10	171 -100
49+2=	49-2=	65 -5	75 +100	93-10=	75+25=
125-25=	198+10=	155 +5	400 -25	297-100=	100-2=

FAST Math +−÷x

Lesson 13:

Counting by 11s, 12s, and 20s

Learning objectives

- Count by 11, 12, and 20

These may seem a bit difficult, but the challenge is not too far a stretch. It is enormously valuable for students to feel that their minds have the power to do things that are not easy. These numbers have been selected because of their general utility in life's counting situations.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Gather materials specific to this lesson:
  - Paper bag and fist full of real or pretend dimes (for each pair of students)
  - Six 12-inch rulers
  - Egg cartons (at least 6)
- Print the PDFs for Lesson 13:
  - Blank 100 Number Grid (1 for each student)
  - Fist Full of Coins Game Card (or reuse from Lesson 12)
  - $20 Bills (print and cut out)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, *"Time for Warmup."*

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by 11s, helping them "catch" the pattern.

4. Lead students in counting by 12s, from 0–72. Then, for additional practice, count by 12s from 100–172, 200–272, and 300–372.

Note:

It may help at first to take it slow and look at the number line as you lead this counting activity.

5. Lead students in counting by 20s. Keep counting in this pattern up to 1,000.

Round Robin

6. Conduct the Round Robin, first by 11s, then by 12s (to 96), and then by 20s (to 1,000).

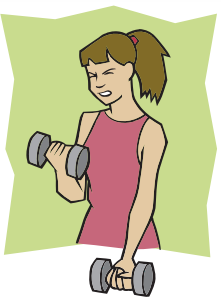
Number Grid

7. Have students count by 12s and circle the multiples of 12 on their individual number grids. Then have them put squares around counting by 11s and triangles on counting by 20s.

8. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout

Complete the transition to Workout routine devised in Lesson 1.



Workout

(10–15 minutes)

Talk: Counting by 11s and 12s

1. Lead a discussion about how you can "think" about counting by 11s (counting by 10 and adding one more). Demonstrate this process on the number line. Have students talk about the pattern they hear counting when counting by 11s.

2. Demonstrate on the number line how to count by 12s (counting by 10 and adding two more). Then discuss ways that counting by 12s can be useful. Elicit examples from students if they have any.

3. Distribute the 12" rulers and have the students put them end-to-end, counting by 12s (and being sure to have the students verbalize 12 inches, 24 inches, 36 inches, etc.)

4. Then lay out the egg cartons and have students count by 12s, verbalizing the thinking process (counting 10 and adding 2 more). Repeat this counting by 12s until it becomes easier for students.

5. Also, be sure that the students understand that "a dozen" is another way of referring to 12. Have them use the term repeatedly, so it sticks (1 dozen, 2 dozen, 3 dozen, etc.).

Explore: Counting by 20s

6. Talk together about counting by 20s. Using the number line, figure out how many 20s make 100.

7. Count twenty-dollar bills with the students.

8. Pull out the bag of dimes and demonstrate the connection between counting pairs of dimes and counting by 20s.

9. Use the Fist Full of Coins Game Card to play Fist Full of Coins with dimes only. Have students count up their dimes 2-at-a-time, counting by 20s. (When they reach 100, be sure they say both "1 hundred" and "1 dollar." Do the same for 2 dollars and 3 dollars.)

One-Minute Worksheet

10. Provide the One-Minute Worksheet and time students for 60 seconds. Remind students that they can think about what they know to solve the problems.

11. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 1,101–1,200. Be sure to help them do this correctly. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on counting by 11s, 12s, and 20s.

- *What's easy?*
- *What's not so easy?*

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Fist Full of Coins Game Card

Variations:
The game can be played with one type of coin as well as mixed coins. Initially, have the students play with one type of coin (nickels, dimes, or quarters) as appropriate to the lesson. Starting with Lesson 12, mix the coins together.

Fist Full of Coins

Number of Players: 2

Materials:
- **coins in a bag**

Each player reaches into the paper bag (without peeking) and grabs a small fistful of coins. Players count the value of their coins. The player with the higher value wins the round and keeps his/her coins. The other player returns his/her coins to the bag. If a player catches an opponent miscounting, that player wins the round. Play until the bag is empty. The player with the most coins at the end of the game wins.

FAST Math +−÷x

Name: ____________________ Date: ____________________

$20 Bills

$20 *not legal tender* $20

$20 **TWENTY DOLLARS** $20

$20 *not legal tender* $20

$20 **TWENTY DOLLARS** $20

$20 *not legal tender* $20

$20 **TWENTY DOLLARS** $20

$20 *not legal tender* $20

$20 **TWENTY DOLLARS** $20

$20 *not legal tender* $20

$20 **TWENTY DOLLARS** $20

Name: _____ Date: _____

Lesson 13: One-Minute Worksheet

11+11=	12+12=	20 +20	22 +11	24+12=	40+20=
33 +11	36 +12	60+20=	44+11=	48 +12	80 +20
55+11=	60+12=	100 +20	66 +11	72+12=	120 +20
77 +11	84 +12	140 +20	88 +11	160+20=	11+11=
12+12=	180+20=	22 +11	24 +12	200+20=	33+11=
36 +12	220 +20	44+11=	48+12=	240 +20	55 +11
60+12=	260+20=	66 +11	72 +12	280+20=	77+11=
84 +12	180 +20	11+11=	36+12=	80 +20	66 +11
160 +20	44 +11	72+12=	260+20=	22 +11	24 +12
280 +20	140 +20	77 +11	84 +12	180 +20	99 +11

FAST Math +−÷×

Lesson 14:

Estimating

Learning objectives

- Round numbers to the nearest 10
- Use rounding to estimate in advance and to check after calculating

By this point, students have developed a robust repertoire of skills on the visible and their mental number lines. They have developed their understanding of the number system in important ways. In this lesson they are directed to use those skills and understandings to monitor their math performance by estimating and checking. Such monitoring is a crucial habit to develop on the path to more complex math concepts and procedures.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 14:
  - Filled-in 100 Number Grid (1 for each student, if needed)
  - Blank 100 Number Grid (1 for each student)
  - One-Minute Worksheet (1 for each student)
- Copy the PDFs so that there are enough copies for each student

Warmup

(5–10 minutes)

1. Have the student of the day place the clothespin on the Warmup icon of the lesson chart and announce, "*Time for Warmup*."

Opening Exercise

2. Conduct the 30-second Opening Exercise you chose from the suggestions in Lesson 1 to get everyone cooperating in unison.

Count Together

3. Lead students in counting by whatever numbers or combinations they need more practice with. If appropriate, let a student lead these count-alouds.

Round Robin

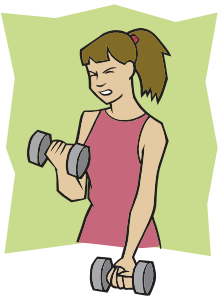
4. Conduct the Round Robin, using whatever count sequence the students need more practice with. If appropriate, let a student lead these count-alouds.

Number Grid

5. Have students mark any counting sequence on the number grid that the students may need more practice with. Otherwise, do not include number grid work for this lesson.

6. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Workout.

Transition to Workout
Complete the transition to Workout routine devised in Lesson 1.



Workout
(10–15 minutes)

Tip:

Math Vocabulary
Math vocabulary is not a trivial matter! Students need to actively use unfamiliar terms. Teachers often remember to use new terms themselves; they often forget to have their students use them—which is far, far more crucial for learning.
Benefits of using math vocabulary include:

- Physically articulating the word helps fix its location in a student's mind
- Repeated recalling strengthens the mental route to the word
- Using it reinforces *how to* use it appropriately

Talk and Explore: Estimating

1. Lead a discussion about how students can now "think" + or - 10. Point out that this is very strong math thinking.

2. Propose the idea that "thinking + or - 10" can help them estimate answers or how to figure out about what an answer might be. Throughout the discussion, use the terms *estimate*, *estimating*, *estimation*—and then be sure that the students use these terms.

Provide an example, such as:
I had 3 items in my shopping cart and wanted to add them up in my head to see if I had enough money. The items cost $8 and $13 and $11. So, I estimated by "thinking" 8 is close to 10, 13 is close to 10, and 11 is close to 10. 10+10+10 is 30, so these will cost about $30.

On the number line, demonstrate that 8 is close to 10 (as are 13 and 11). Then calculate the exact answer (on the chalkboard or with a calculator) and point out that the answer is a good estimation of the exact answer.

Provide another example:
If the things I wanted to buy cost $43 and $32 and $21, can we figure out how to estimate an answer?

Using a number line to provide a visual example, put paperclips on 43, 32, and 21. Demonstrate that 43 is close to 40, 32 is close to 30, and 21 is close to 20. Then think aloud, "*40 and 30, that's 40, 50, 60, 70, plus 20 more—80, 90—so, that's about $90.*" Then calculate the exact answer (on the chalkboard or with a calculator) and point out that the answer is a good estimation of the exact answer.

3. Provide other examples for the students to mark on their number lines with paperclips. When they have estimates, let them check exact answers with a calculator. Be sure they talk through their thinking, explaining each estimation and calculation step, using the correct vocabulary: estimate, estimating, and estimation.
   - *What if the items cost $7 and $24 and $9?*
   - *What if the items cost $47 and $49 and $58?*
   - *What if the items cost $73 and $26 and $12?*

One-Minute Worksheet

4. Provide the One-Minute Worksheet and time students for 60 seconds. Remind students that they can think about what they know to solve the problems.

5. Have the student of the day move the clothespin on the lesson chart and announce the beginning of Cooldown.

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Have each student fill in a Blank Number Grid from 1,201–1,300. Be sure to help them do this correctly, counting aloud the numbers they are writing. Add the completed grids to the collection started in Lesson 1.

Reflect

2. Reflect together on the usefulness of estimating.

Paperclips

3. Follow the steps in Lesson 1 to add another 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Name: _____ Date: _____

Filled-in 100 Number Grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FAST Math +−÷x

Name: _____ Date: _____

Blank 100 Number Grid

FAST Math +−÷x

Name: _____ Date: _____

Lesson 14: One-Minute Worksheet

19 + 14 □ + □	24 + 17 □ + □	26 + 18 □ + □	13 + 71 □ + □
29 + 28 □ + □	21 + 37 □ + □	39 + 43 □ + □	52 + 66 □ + □
41 + 46 □ + □	49 + 48 □ + □	52 + 61 □ + □	33 + 44 □ + □
32 + 48 □ + □	56 + 32 □ + □	38 + 49 □ + □	52 + 48 □ + □
63 + 53 □ + □	67 + 48 □ + □	68 + 67 □ + □	77 + 88 □ + □
56 + 64 □ + □	81 + 93 □ + □	96 + 97 □ + □	97 + 98 □ + □
32 + 98 □ + □	47 + 83 □ + □	43 + 87 □ + □	52 + 26 □ + □
91 + 92 □ + □	83 + 84 □ + □	74 + 77 □ + □	71 + 63 □ + □
78 + 82 □ + □	98 + 98 □ + □	87 + 97 □ + □	87 + 98 □ + □
91 + 89 □ + □	66 + 29 □ + □	72 + 87 □ + □	99 + 94 □ + □

FAST Math +−÷×

Lesson 15:

Counting Olympics

Learning objective

- Celebrate students' progress and achievement by having them compete for medals at Olympic Counting events at four stations

This finale reformats the lesson into a set of Olympic events, in which students' training culminates in a demonstration of skill and medal garnering. Your young math athletes should, rightfully, feel a sense of accomplishment. Their muscular counting and nimble mental gymnastics are the results of their efforts and persistence. This event is an opportunity for them to show how far their skills have developed and to celebrate their mental prowess.

Teacher preparation

- Gather the materials needed for all lessons (see list on page 40)
- Print the PDFs for Lesson 15 which include:
  - Olympic Event Score Cards (1 for each student)
  - Judge's Sheet
  - Gold and Silver Medals (several copies)
- Copy the PDFs so that there are enough copies

This final session is organized into Olympic events, so the usual Warmup, Workout, Cooldown regime is set aside. Instead, the lesson is set up as four simultaneous Olympic Event Stations followed by a cooldown and closing ceremony.

Getting Ready

1. Set up four stations for the Olympic Counting events. These can include signs or banners and a list of events at each station. You can use the classroom, but it might be exciting to do this in the gym or some other space.

A Big Celebration
If several groups of students have been working through these lessons, then the Counting Olympics can be an even more special event, with more participants and much hubbub. For example, if there is a group of students using the lessons in every classroom across the grade level, you might want to organize a larger Counting Olympics with the other classes. This can be a big celebration and promote students' feeling particularly acknowledged for their achievements.

2. Put a station-master (a teacher, student from another class, or administrator) in charge of each station to administer tasks and mark students' scorecards. (If there is only one station-master, then students can all move from station to station together.)

3. Explain to students how the Counting Olympics events will work:
   - *Each Counting Athlete performs all events at each of the four stations*
   - *Every athlete will have a scorecard for recording their performance*
   - *When students finish at one station, their scorecards get marked by the station-master, and they move to the next station*
   - *Explain how students will move from station to station—the rotation order*
   - *At the end of all events, athletes receive medals based on their scores*

4. Divide students and assign them to their first station. As students complete the events at one station, they get their scorecards marked for each event that they complete accurately and efficiently and proceed to the next station. The station-master can guide the athletes through the tasks one at a time, but the more students can perform on their own, the better. The scorecard should be marked as follows:

Medal	Performance
GOLD	Events performed accurately, nimbly, and with assurance
SILVER	Events performed accurately but slowly or with hesitations or self-corrections
NO MEDAL	Events that are inaccurate

5. After completing all events, students collect their medals and settle down together to celebrate their math achievements and conclude with a final Cooldown routine.

The Counting Olympics Stations

The four stations and events are outlined below:

Station 1: Counting (10 Events)
Station 2: Addition and Subtraction (5 Events)
Station 3: Money (6 Events)
Station 4: Math Demonstrations (7 Events)

The Counting Station (10 Events)

Event 1: Counting-on by 1s from anywhere
Event 2: Counting the decades, forward and back
Event 3: Counting by 5s (in the five scale)
Event 4: Counting by 2s (even) from different starting numbers
Event 5: Counting by 2s (odd) from different starting numbers
Event 6: Counting by 25s
Event 7: Counting by 100s forward and back
Event 8: Counting by 11s
Event 9: Counting by 12s
Event 10: Counting by 20s

The Addition and Subtraction Station (5 Events)

Event 1: + and - 10 from any number
Event 2: + and - 5 from any number on the 5-scale
Event 3: + and - 2 from any even number
Event 4: + and - 2 from any odd number
Event 5: + and - 25 from any number in the 25 count sequence

The Money Station (6 Events)

Event 1: 4 quarters =__; 2 quarters = __; 3 quarters =__; 5 quarters =__
Event 2: How many $20 bills make $100?
Event 3: How much is 10 dimes? How much is 5 nickels?
Event 4: How many $20 bills make $200?
Event 5: How many quarters make $5?
Event 6: How much is 4 pennies, 3 nickels, 4 dimes, and 1 quarter?

The Math Demonstrations Station (7 Events)

Event 1: On the number line, show counting by 10 starting with 28.
Event 2: On the number line, show counting backwards by 10 starting with 92.
Event 3: What are all the math words for moving forward on the number line? (add, adding, addition, plus, and)
Event 4: What are all the math words for moving backwards on the number line? (subtract, subtracting, subtraction, minus, take away)
Event 5: Say whether these numbers are odd or even: 99, 84, 77, 66, 54, 45, 32, 21, 13, 8
Event 6: On a number grid with only 0 and 100 written on it, show where the following numbers would go (without counting every space): 10, 40, 80, 35, 55, 75, 29, 69, 99, 34, 67
Event 7: Show how you would estimate the answers to these problems: 12+38; 97+36; 68+78

Transition to Cooldown
Complete the transition to Cooldown routine devised in Lesson 1.

Cooldown

(5 minutes)

Number Grid

1. Give students their accumulated sequence of Number Grids. If it seems appropriate, they can staple them one to another in a sequence to see how LONG they are when they're all connected (1,000+ numbers makes a very long scroll!). Then, they can roll them up, put a rubber band around them, and take their scroll home as a kind of diploma.

Reflect

2. Reflect on all the exercising students have done and on their strong mental math powers. It might be helpful to ask students to list all the types of counting that they can now do with ease.

Paperclips

3. Follow the steps in Lesson 1 to add the final 100 paperclips to the Great Ongoing Accumulating Paperclip Collection.

Closing Ceremony

4. Do something celebratory to mark the conclusion of the Counting System lessons. You might do any combination of the following (or use your own ideas):

- Do a few rounds of the Opening Exercise
- Take a vote on students' favorite count sequence and chant that Count Together sequence
- Divide up the 10-strings of paperclips equally among the students (of course, ensure that the students count the number they are carting home)
- Ask students how they'd like to bring this good work to a conclusion

Name: _____ Date: _____

Olympic Event Score Card

Olympic Event Score Card

Athlete Name: _____ **Event Date:** _____

	GOLD	SILVER	NONE	INITIALS
COUNTING				
Event 1	☐	☐	☐	_____
Event 2	☐	☐	☐	_____
Event 3	☐	☐	☐	_____
Event 4	☐	☐	☐	_____
Event 5	☐	☐	☐	_____
Event 6	☐	☐	☐	_____
Event 7	☐	☐	☐	_____
Event 8	☐	☐	☐	_____
Event 9	☐	☐	☐	_____
Event10	☐	☐	☐	_____
ADDITION & SUBTRACTION				
Event 1	☐	☐	☐	_____
Event 2	☐	☐	☐	_____
Event 3	☐	☐	☐	_____
Event 4	☐	☐	☐	_____
Event 5	☐	☐	☐	_____
MATH DEMONSTRATIONS				
Event 1	☐	☐	☐	_____
Event 2	☐	☐	☐	_____
Event 3	☐	☐	☐	_____
Event 4	☐	☐	☐	_____
Event 5	☐	☐	☐	_____
Event 6	☐	☐	☐	_____
Event 7	☐	☐	☐	_____
MONEY				
Event 1	☐	☐	☐	_____
Event 2	☐	☐	☐	_____
Event 3	☐	☐	☐	_____
Event 4	☐	☐	☐	_____
Event 5	☐	☐	☐	_____
Event 6	☐	☐	☐	_____

FAST Math +−÷×

Name: _____ Date: _____

Judge's Sheet

The Counting Station

Directions for the Station-Master:
As each Olympic athlete visits the station, have them demonstrate their counting abilities by completing each event below. At the end of each event, use the medal criteria and mark the athlete's score card.

The Counting Station

Event 1: Counting-on by 1s from anywhere

Event 2: Counting the decades, forward and back

Event 3: Counting by 5s (in the five scale)

Event 4: Counting by 2s (even) from different starting numbers

Event 5: Counting by 2s (odd) from different starting numbers

Event 6: Counting by 25s

Event 7: Counting by 100s forward and back

Event 8: Counting by 11s

Event 9: Counting by 12s

Event 10: Counting by 20s

Medal	Performance
GOLD	events performed accurately, nimbly, and with assurance
SILVER	events performed accurately but slowly or with hesitations or self-corrections
NO MEDAL	events are inaccurate

FAST Math +−÷x

Name: _____ Date: _____

Judge's Sheet

The Addition and Subtraction Station

Directions for the Station-Master:
As each Olympic athlete visits the station, have them demonstrate their counting abilities by completing each event below. At the end of each event, use the medal criteria and mark the athlete's score card.

The Addition and Subtraction Station

Event 1: + and - 10 from any number

Event 2: + and - 5 from any number on the 5-scale

Event 3: + and - 2 from any even number

Event 4: + and - 2 from any odd number

Event 5: + and - 25 from any number in the 25-count sequence

Medal	Performance
GOLD	events performed accurately, nimbly, and with assurance
SILVER	events performed accurately but slowly or with hesitations or self-corrections
NO MEDAL	events are inaccurate

FAST Math +−÷x

Name: _____ Date: _____

Judge's Sheet

The Money Station

Directions for the Station-Master:
As each Olympic athlete visits the station, have them demonstrate their counting abilities by completing each event below. At the end of each event, use the medal criteria and mark the athlete's score card.

The Money Station

Event 1: 4 quarters = ___ ; 2 quarters = ___ ; 3 quarters = ___ ; 5 quarters = ___

Event 2: How many $20 bills make $100?

Event 3: How much is 10 dimes? How much is 5 nickels?

Event 4: How many $20 bills make $200?

Event 5: How many quarters make $5?

Event 6: How much is 4 pennies, 3 nickels, 4 dimes, and 1 quarter?

Medal	Performance
GOLD	events performed accurately, nimbly, and with assurance
SILVER	events performed accurately but slowly or with hesitations or self-corrections
NO MEDAL	events are inaccurate

FAST Math +−÷x

Name: _____ Date: _____

Judge's Sheet

The Math Demonstrations Station

Directions for the Station-Master:
As each Olympic athlete visits the station, have them demonstrate their counting abilities by completing each event below. At the end of each event, use the medal criteria and mark the athlete's score card.

The Math Demonstrations Station

Event 1: On the number line, show counting by 10 starting with 28.

Event 2: On the number line, show counting backwards by 10 starting with 92.

Event 3: What are all the math words for moving *forward* on the number line? (add, adding, addition, plus, and)

Event 4: What are all the math words for moving backwards on the number line? (subtract, subtracting, subtraction, minus, take away)

Event 5: Say whether these numbers are odd or even: 99, 84, 77, 66, 54, 45, 32, 21, 13, 8

Event 6: On a number grid with only 0 and 100 written on it, show where the following numbers would go (without counting every space): 10, 40, 80, 35, 55, 75, 29, 69, 99, 34, 67

Event 7: Show how you would estimate the answers to these problems: 12+38; 97+36; 68+78

Medal	Performance
GOLD	events performed accurately, nimbly, and with assurance
SILVER	events performed accurately but slowly or with hesitations or self-corrections
NO MEDAL	events are inaccurate

FAST Math +−÷x

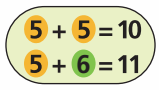
Name: _____ Date: _____

Gold and Silver Medals

Copy the medals onto heavy card stock, color, and cut out. Hang the medals from string or ribbon and give them to students at the end of the Counting Olympics.

FAST Math +−÷x

Number-Fact Linking Intervention



Strengthening connections between math facts

Some students do not shift from primitive to mature strategies for basic number facts. Instead, they get stuck using early strategies that are slow and attention-intensive. When adding, for example, they may count-all instead of shifting to counting on from the higher number—solving 4+5 by counting up to 4 and then up 5 more. Or they may count-on, not progressing to linking from known facts (e.g., leaping in one mental move from the known 4+4=8 to solve 4+5). When multiplying, they may laboriously count-up for problems like 5x6, instead of using the well-known 5x5=25 as a stepping-stone. Unlike students with well-developed fact-linking strategies, they do not use efficient shortcuts that reduce the number of mental moves needed to figure answers.

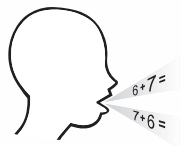
We all use a mix of counting and linking strategies, as well as direct retrieval, when we reach into our minds for math facts in all four operations. Those of us who are fast with our facts access more of them directly from memory, but we also activate well-developed networks of fact linkages that we navigate strategically. Rich networks of interconnected facts are important, for monitoring accuracy in our math performance and for calling on as backups when direct retrieval falters, for whatever reason (e.g., fatigue, summertime disuse, fluctuations in memory or attention, etc.)

Students develop—usually on their own—a variety of links among number facts, as their minds discover shortcuts from years of computing combinations. This natural outgrowth of prolonged practice can be stimulated and strengthened through instruction that points out and practices strategies for linking well-known and lesser-known number facts.

Features of intervention

Below are features of effective intervention for developing students' fact-linking strategies in addition, subtraction, multiplication, and division.

Practice aloud and in writing



Mastering each fact group means answering readily and with assurance, no matter the format. So, it is important to provide practice:

- Orally—with each number in the first position

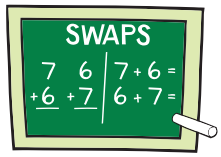
and

- In written numerals—presented both horizontally and vertically, as well as with each numeral written in the first position

Orally presented practice can be particularly important to securing facts in memory, so be sure to pay particular attention to this multi-sensory aspect in your instruction and preparation of practice materials.

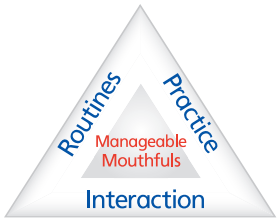
Emphasize "swaps"

Another critical feature of this intervention is the commutative principle. Commutatives, or "swaps" (e.g., 8+7 and 7+8, 5x6 and 6x5), need emphasis for each fact group that is practiced. Without this emphasis, students commonly conceive of swaps as two different facts, effectively doubling the number of facts they need to master. Securing a firm grasp on the commutative principle means acquiring a fundamental mathematical understanding.



Provide manageable mouthfuls, interaction, and frequent practice

Establishing well-known facts and linking these to related facts does not need to involve lengthy intervention sessions, but it does require systematic practice with small groups of facts. These are best carried out in brief, frequent sessions—ideally, five times per week for intensive 15-minute sessions or three times per week for 20–30 minutes.



Effective intervention involves teacher-led, interactive, and highly practiced instructional routines that maintain students' engagement for brief sessions. In addition, effective intervention intersperses practicing the target group of facts with cumulative practice that mixes groups of previously mastered facts.

Follow the progression of number fact groupings

In the following section you will find an instructional sequence for the Number-Fact Linking Intervention, specifying what is to be taught, in what groupings, and according to what order. The groupings within each operation build one on another. And the operations also follow a sequence: mastery of addition must precede subtraction or multiplication, and a foundation in multiplication is essential before addressing division.You will also find a suggested instructional routine that you can readily modify to fit the age and specific abilities of your students.

The following interventions are not laid out in "lessons," but rather in a progression of fact groups to be worked on until they are mastered. The initial groups become the well-known facts to be linked to lesser-known facts, therefore, mastery is important before moving on to each subsequent group.

Use this intervention along with the FASTT Math software

The Number-Fact Linking Intervention can be used simultaneously with the *FASTT Math* software. The fact linking lessons provide backup strategies that support students as they acquire math fact automaticity.

The intervention outlines below provide you with:

- A suggested instructional routine
- What is to be taught
- In what groupings
- According to what sequence

Teacher preparation

Use the *Math Fact Diagnostic Assessment* to determine if the student needs work on addition/subtraction or multiplication/division. Before beginning the Number-Fact Linking Intervention, make sure the student has a firm grasp of quantity concepts and counting.

Print and assemble:

- Print out the PDF for the target operation. Make a copy of each worksheet for each student. You will find four types of worksheets:
  1. Fact Group Practice Pages (40 problems from the fact group under study)
  2. Mixed Fact Group Practice Pages (40 problems from the current fact group and related base facts)
  3. Think Pages (28 problems to identify the linking fact, not the answer)
  4. One-Minute Progress Checks (40 fact group problems to complete in one minute)
- Make Number Fact Cards for each fact group. (Print the Fact Group Practice Pages on heavy paper stock and cut them out, or copy the content onto index cards.)

Accessing the PDF files

The PDF files for the Number-Fact Linking Intervention can be accessed through the teacher administration tool (see page 4 for instructions).

Note:

See page iv for information about the teacher administration tool.

Review the fact groupings: Base facts & related facts

The following number fact groupings should be taught sequentially. The base facts—the facts students know—must be learned thoroughly so that students can "link" the related facts to them. Within addition, the Doubles (2+2, 3+3, and so on) and +10 (adding 10 to any number) serve as the main base facts (*I know that 4+4 = 8 and 4+5 is just one more, or 9.)* In multiplication the emphasis is on swaps (*6x5 is the same as 5x6*).

Progression of number fact groupings for addition and subtraction

Important!

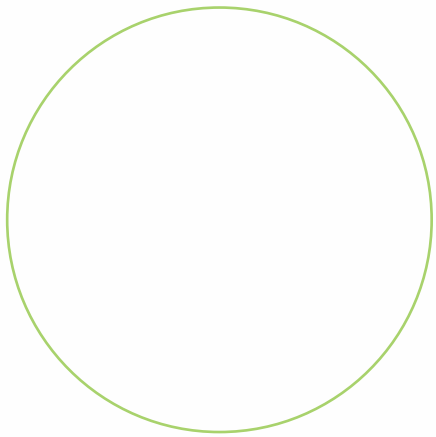
Students should master the entire progression of addition facts, one grouping at a time, and then proceed to the sequence of subtraction fact groups.

Progression of number fact groupings for multiplication and division

When taught in this sequence, many of the more difficult multiplication tables never need to be "memorized" as such, since they are included as part of this progression. For example, there's no need for the "7 table" when so many of 7 facts have already been learned with the base facts (7x0, x1, x2, x5, x9, x10)…if you emphasize swaps.

Important!

Students should master the entire progression of multiplication facts, one grouping at a time, and then proceed to the sequence of division fact groups.



Follow the Sample Routine

Plan for brief, frequent sessions—ideally, five times per week for intensive 15-minute sessions or three times per week for 20–30 minutes. The instructional routine below is designed to reinforce known, or base facts and then build links to related facts. Feel free to modify it to suit circumstances and your students' needs.

The progression of Number Fact Groupings on the following pages separates "base" facts from their related facts. The base facts must be well established before students can link them to answer new facts. In multiplication, the "swaps" are the related facts.

Lesson characteristics

Instructional sessions need to be:

- Engaging
- Highly focused
- Interactive
- Up-tempo (note the short time interval for each activity—use a timer to keep things moving)

Sample Routine

Follow the suggested lesson routine below for each group of facts.

1. Run-through (2–3 minutes)

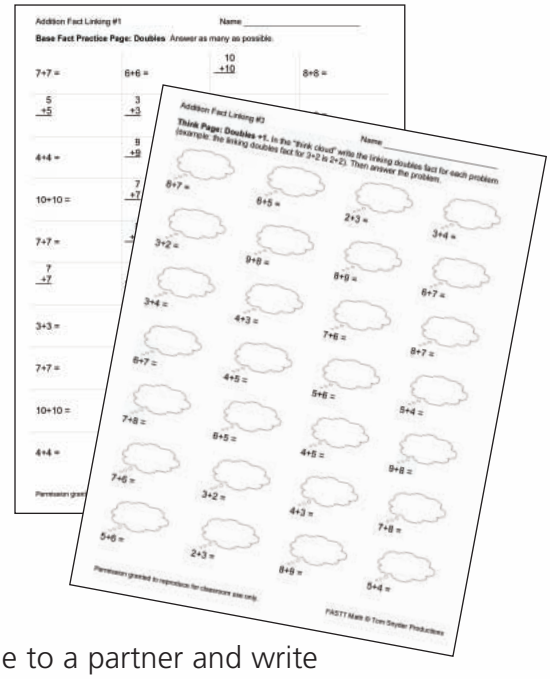
   Working with one student at a time (or two students working together in a pair), run through the fact group card pack. As the student answers each fact problem, sort the cards into those that are "fast" and those that are "slow." Put the "fast" cards aside, and do a second run-through with the facts that are "slow."

2. Talk about it (2–3 minutes)

   Discuss ways to help recall the facts of this grouping. For the Doubles group of base facts, the Last 10 Multiplication Facts, and for any particularly stubborn facts, some of these memory supports could be:

   - Concrete mnemonics
   - Use rhymes or songs
   - Practice saying a difficult fact over and over
   - Practice writing a difficult fact over and over, each time on one side of a separate card with the answer on the back
   - Write a difficult fact on a piece of paper, fold it up, and carry it in your pocket all day (have others ask you, "*Which number fact is in your pocket?*")
   - Have someone write a difficult fact/answer on your back with their finger

For the other fact groups, the discussion should relate directly to the thinking strategy for linking one fact to its base partner (e.g., for 6+7 think 6+6, for 9+6 think 10+6).



3. Workpages (2 minutes)

Print the PDF worksheets for the operation that your student needs practice with. The worksheets can be used as many times as needed to give students practice with the number fact groupings. Use the Fact Practice pages and the Think pages in a variety of creative ways, for example:

- Students can swap Think pages after they've filled in the clouds with the linking fact. Students can check each other's pages and then write the answers on their partner's sheet
- Students can circle all the harder facts and practice those
- Students can read the problems one at a time to a partner and write down their partner's answers, noting ✓ for correct, + for correct-and-fast
- Students can see how many facts they can complete in 30 seconds and then choose only the harder remaining ones to practice
- Students can practice the whole page in quiet concentration if that will help secure the more difficult facts in that fact group

4. Partner Practice (4–5 minutes)

Using the Number Fact Cards (vertical and horizontal problems without answers) for each grouping of facts, students can simply drill one another, putting the "slower" facts in a practice-again pile and drilling them until they seem to be sticking.

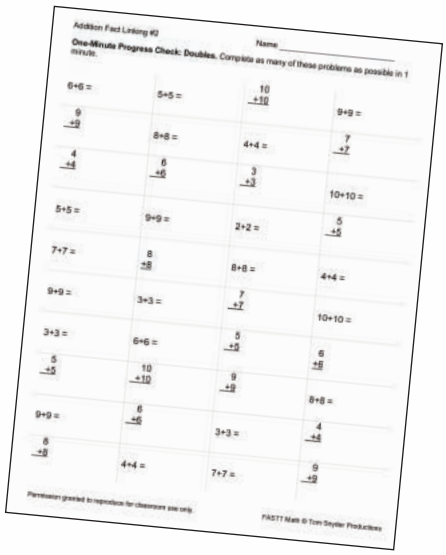
Tip:

Partner Practice Games

Alternatively, create various 2-player games. Be sure they are focused and provide lots of active practice. Some ideas include:

- WAR: Split the Number Fact Cards between the two players, play a traditional card war with each player flipping a card and answering the fact (higher answer takes both). When they each flip the same problem, players then flip three cards each, answer the fact, with the higher final flip taking all.
- BUDDY: Each player brings his or her own deck of Number Fact Cards to the game. Each player takes 4 cards and puts the remaining cards in a pile. Player 1 reads aloud one of his or her cards (e.g., "I have 4+6= (10) do you have my buddy, 6+4=(10)?"). Player 2 hands over the buddy if he or she has it. Each buddy pair is laid face up. Players take turns until all the buddies are found.

5. One-Minute Progress Check (1 minute)


Addition Fact Linking #2 Name _____

One-Minute Progress Check: Doubles. Complete as many of these problems as possible in 1 minute.

6+6 =	5+5 =	10 +10	9+9 =
9 +9	8+8 =	4+4 =	7 +7
4 +4	6 +6	3 +3	10+10 =
5+5 =	9+9 =	2+2 =	5 +5
7+7 =	8 +8	8+8 =	4+4 =
9+9 =	3+3 =	7 +7	10+10 =
3+3 =	6+6 =	5 +5	6 +6
5 +5	10 +10	9 +9	8+8 =
9+9 =	6 +6	3+3 =	4 +4
6 +6	4+4 =	7+7 =	9 +9

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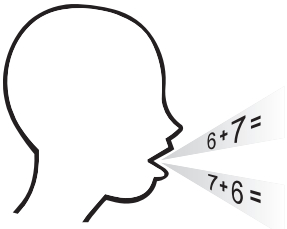
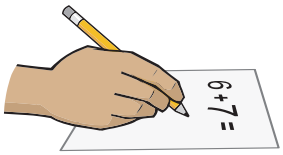
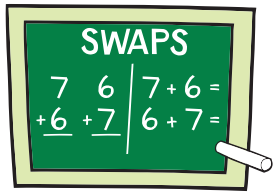
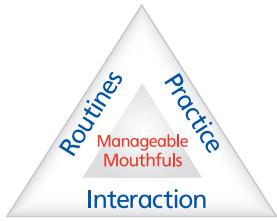
Keep a progress chart that shows the number correct in one minute for each student for the fact group. This is an important progress indicator for instructional decision-making, as well as a potent motivator for students to keep practicing.

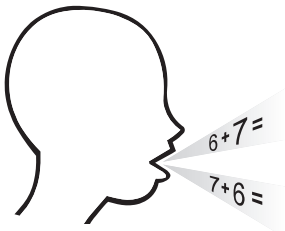


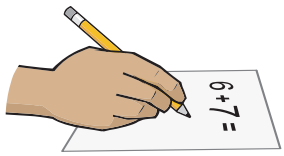


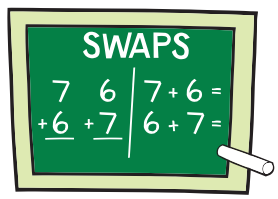


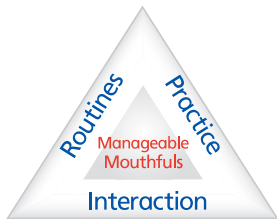


Addition & Subtraction Number Fact Groupings: (Work on one group at a time to master)	**Includes:**	
Doubles (base fact group) It is likely that your students are speedy with some of these already. The aim is to get all the doubles facts up to speed, with each student knowing them—snap!—when presented orally and in writing (vertically and horizontally).	**ADDITION** *2+2 through 10+10*	**SUBTRACTION**
	2+2 3+3 4+4 5+5 6+6 7+7 8+8 9+9 10+10	4-2 6-3 8-4 10-5 12-6 14-7 16-8 18-9 20-10
Doubles +1 **Link to Doubles base facts (think Doubles and one more)** This strategy is necessarily slow at first. Take the time needed to ensure that students are "thinking" and "envisioning" each Doubles +1 problem as Doubles and one more. Once they can "think" this way, performance will speed up.	*2+3 through 9+10*	
	2+3 and 3+2 3+4 and 4+3 4+5 and 5+4 5+6 and 6+5 6+7 and 7+6 7+8 and 8+7 8+9 and 9+8 9+10 and 10+9	5-2 and 5-3 7-3 and 7-4 9-4 and 9-5 11-5 and 11-6 13-6 and 13-7 15-7 and 15-8 17-8 and 17-9 19-9 and 19-10
Doubles +2 **Link to Doubles base facts (think Doubles and two more)** Follow the same instructions as those under Doubles +1.	*2+4 through 8+10*	
	2+4 and 4+2 3+5 and 5+3 4+6 and 6+4 5+7 and 7+5 6+8 and 8+6 7+9 and 9+7 8+10 and 10+8	6-2 and 6-4 8-3 and 8-5 10-4 and 10-6 12-5 and 12-7 14-6 and 14-8 16-7 and 16-9 18-8 and 18-10
+1 and +0 These are not strictly number facts, but rather concepts that can be applied to any number. The point of including them here is to ensure that there is no confusion around them.	to numbers 0–20 (then 0–100, then 0–1,000)	
+10 (base fact group) This is also not strictly a number fact, but a concept that can be applied to any number. It is included here as a linking strategy for the +9 problems that follow.	to numbers 0–9 (then 0–100)	

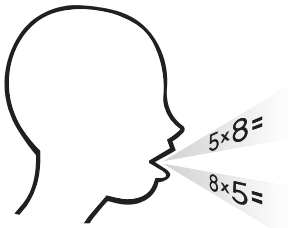


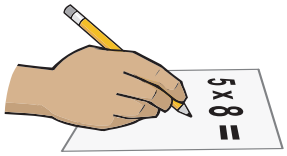


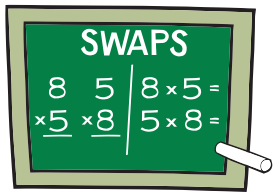


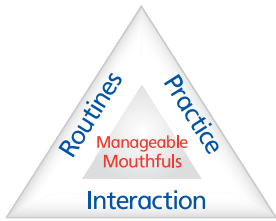


Addition & Subtraction Number Fact Groupings: (cont.)	Includes:	
+9 **Link to +10 base facts (think +10 minus one)** This is also not strictly a number fact, but a concept that can be applied to any number. It is included here as a linking strategy for the +9 problems that follow.	to numbers 0–9 (then 0–100)	
Last 10 Addition Facts Here are the remaining facts. The number fact grouping leaves only two handfuls of isolated facts for students to master.	2+5 and 5+2 2+6 and 6+2 2+7 and 7+2 2+8 and 8+2 3+6 and 6+3 3+7 and 7+3 3+8 and 8+3 4+7 and 7+4 4+8 and 8+4 5+8 and 8+5	7-2 and 7-5 8-2 and 8-6 9-2 and 9-7 10-2 and 10-8 9-3 and 9-6 10-3 and 10-7 11-3 and 11-8 11-4 and 11-7 12-4 and 12-8 13-5 and 13-8

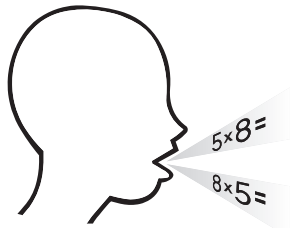


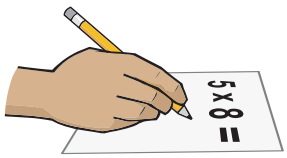


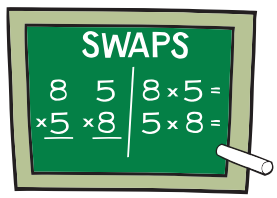


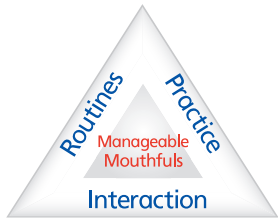


Multiplication & Division Number Fact Groupings: (Work on one group at a time to master)	Includes:	
x2 (base facts) and swaps Emphasize the swaps—3x2 is the same as 2x3. These multiplication facts become the base facts for the related division facts. Help students see the relationship between the division fact (6÷2=__) and the familiar multiplication fact (2x__=6).	**MULTIPLICATION** *2x2 through 2x10*	**DIVISION**
	2x2 2x3 and 3x2 2x4 and 4x2 2x5 and 5x2 2x6 and 6x2 2x7 and 7x2 2x8 and 8x2 2x9 and 9x2 2x10 and 10x2	4÷2 6÷3 8÷4 10÷5 12÷6 14÷7 16÷8 18÷9 20÷10
x5 (base facts) and swaps Emphasize the swaps—4x5 is the same as 5x4. These multiplication facts become the base facts for the related division facts. Help students see the relationship between the division fact (20÷5=__) and the familiar multiplication fact (5x__=20).	*5x2 through 5x10*	
	5x2 and 2x5 5x3 and 3x5 5x4 and 4x5 5x5 5x6 and 6x5 5x7 and 7x5 5x8 and 8x5 5x9 and 9x5 5x10 and 10x5	10÷5 and 10÷2 15÷5 and 15÷3 20÷5 and 20÷4 25÷5 30÷5 and 30÷6 35÷5 and 35÷7 40÷5 and 40÷8 45÷5 and 45÷9 50÷5 and 50÷10
x9 (base facts) and swaps Emphasize the swaps—6x9 is the same as 9x6. These multiplication facts become the base facts for the related division facts. Help students see the relationship between the division fact (54÷9=__) and the familiar multiplication fact (9x__=54).	*9x2 through 9x10*	
	9x2 and 2x9 9x3 and 3x9 9x4 and 4x9 9x5 and 5x9 9x6 and 6x9 9x7 and 7x9 9x8 and 8x9 9x9 9x10 and 10x9	18÷9 and 18÷2 27÷9 and 27÷3 36÷9 and 36÷4 45÷9 and 45÷5 54÷9 and 54÷6 63÷9 and 63÷7 72÷9 and 72÷8 81÷9 90÷9 and 90÷10
x1 and x0 These are not strictly number facts, but concepts that can be applied to any number. The point of including them here is to ensure that there is no confusion around them.	to numbers 0–10 (then 0–100, then 0–1,000)	









Multiplication & Division Number Fact Groupings: (cont.)	Includes:	
Perfect Squares No swaps here, just one number times itself. These multiplication facts become the base facts for the related division facts. Help students see the relationship between the division fact (36÷6=__) and the familiar multiplication fact (6x__=36).	*1x1 through 10x10*	
	1X1 2X2 3X3 4X4 5X5 6X6 7X7 8X8 9X9 10X10	1÷1 4÷2 9÷3 16÷4 25÷5 36÷6 49÷7 56÷8 81÷9
Last 10 Multiplication Facts Here are the remaining facts. The number fact grouping leaves only two handfuls of isolated multiplication facts and their swaps for students to master. These multiplication facts become the base facts for the related division facts. Help students see the relationship between the division fact (32÷4=__) and the familiar multiplication fact (4x__=32).	3x4 and 4x3 3x6 and 6x3 3x7 and 7x3 3x8 and 8x3 4x6 and 6x4 4x7 and 7x4 4x8 and 8x4 6x7 and 7x6 6x8 and 8x6 7x8 and 8x7	12÷3 and 12÷4 18÷3 and 18÷6 21÷3 and 21÷7 24÷3 and 24÷8 24÷4 and 24÷6 28÷4 and 28÷7 32÷4 and 32÷8 42÷6 and 42÷7 48÷6 and 48÷8 56÷7 and 56÷8

Addition Fact Linking #1 Name _____

Base Fact Practice Page: Doubles Answer as many as possible.

7+7 =	6+6 =	10 +10	8+8 =
5 +5	3 +3	4+4 =	9+9 =
4+4 =	9 +9	8+8 =	10+10 =
10+10 =	7 +7	6 +6	5 +5
7+7 =	8 +8	9 +9	4 +4
7 +7	3+3 =	5+5 =	5+5 =
3+3 =	7 +7	6 +6	8+8 =
7+7 =	4 +4	9+9 =	8 +8
10+10 =	6+6 =	10 +10	6 +6
4+4 =	10 +10	7 +7	9 +9

Addition Fact Linking #2 Name _____

One-Minute Progress Check: Doubles Complete as many of these problems as possible in 1 minute.

6+6 =	5+5 =	10 +10	9+9 =
9 +9	8+8 =	4+4 =	7 +7
4 +4	6 +6	3 +3	10+10 =
5+5 =	9+9 =	2+2 =	5 +5
7+7 =	8 +8	8+8 =	4+4 =
9+9 =	3+3 =	7 +7	10+10 =
3+3 =	6+6 =	5 +5	6 +6
5 +5	10 +10	9 +9	8+8 =
9+9 =	6 +6	3+3 =	4 +4
8 +8	4+4 =	7+7 =	9 +9

Addition Fact Linking #3 Name _____

Think Page: Doubles +1 In the "think cloud" write the linking doubles fact for each problem (example: the linking doubles fact for 3+2 is 2+2). Then answer the problem.

8+7 = *15*

6+5 =

2+3 =

3+4 =

3+2 =

9+8 =

8+9 =

6+7 =

3+4 =

4+3 =

7+6 =

8+7 =

6+7 =

4+5 =

5+6 =

5+4 =

7+8 =

6+5 =

4+5 =

9+8 =

7+6 =

3+2 =

4+3 =

7+8 =

5+6 =

2+3 =

8+9 =

5+4 =

Addition Fact Linking #4 Name _____

Think Page: Doubles +1 In the "think cloud" write the linking doubles fact for each problem (example: the linking doubles fact for $\overset{3}{+2}$ is $\overset{2}{+2}$). Then answer the problem.

5 +6 *11*	8 +7	4 +3	2 +3	8 +9
4 +5	6 +5	7 +6	5 +4	7 +8
3 +2	3 +4	8 +7	9 +8	6 +7
5 +6	7 +6	8 +9	6 +5	9 +8
6 +7	4 +5	7 +8	5 +4	4 +3

Addition Fact Linking #5 Name _____

Mixed Fact Group Practice Page: Doubles and Doubles +1 Answer as many as possible. Look for linking strategies. For example, for 6+7, think 6+6 plus 1.

6+6 =	6+7 =	7+8 =	8 +8
5 +6	8+9 =	6 +6	9+8 =
4+4 =	8 +7	3+3 =	3 +4
7 +6	2+3 =	4+3 =	6 +5
4+5 =	7 +7	4 +4	7+6 =
5+4 =	6+5 =	10+10 =	6 +7
8 +9	3+2 =	5 +5	9 +8
8+7 =	9 +9	5+6 =	8+8 =
4 +5	5 +4	9+9 =	4 +3
8 +8	3+4 =	7+7 =	7 +8

Addition Fact Linking #6 Name _____

Practice Page: Doubles +1 Link to base facts: doubles.

6+7 =	6+5 =	7+8 =	4 +3
9 +8	3 +2	8 +7	7+6 =
8+7 =	3+4 =	6+5 =	8+9 =
2 +3	5+6 =	9+8 =	3+2 =
4+5 =	8 +9	4+3 =	7 +6
8 +7	6+7 =	4 +5	2 +3
6 +7	7 +8	6 +5	9+8 =
5 +6	4+3 =	7+6 =	5+6 =
2+3 =	8 +9	8 +7	6+7 =
5+4 =	5 +6	5 +4	3 +4

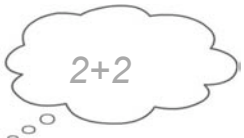
Addition Fact Linking #7 Name _____

One-Minute Progress Check: Doubles +1 Complete as many of these problems as possible in 1 minute.

5 +6	3+4 =	7 +8	4 +3
8+7 =	8 +9	9+8 =	7+6 =
7 +6	5 +4	6+5 =	5+4 =
6 +5	4+5 =	9 +8	5+6 =
3+4 =	7+6 =	8 +7	4 +5
9 +8	7+8 =	4 +3	6 +7
6+7 =	5+6 =	6 +5	8+7 =
7 +6	8+9 =	4+5 =	8+9 =
5 +4	8 +9	6+5 =	9 +8
7+6 =	3 +4	7 +8	4+3 =

Addition Fact Linking #8 Name _____

Think Page: Doubles +2 In the "think cloud" write the linking doubles fact for each problem (example: the linking doubles fact for 6+4 is 4+4). Then answer the problem.



2+4 = 6

7+5 =

7+9 =

5+3 =

4+6 =

9+7 =

3+5 =

9+7 =

6+4 =

4+2 =

6+8 =

5+7 =

5+7 =

4+6 =

8+6 =

3+5 =

7+9 =

7+5 =

4+6 =

7+9 =

4+2 =

5+3 =

9+7 =

8+6 =

6+8 =

2+4 =

5+7 =

6+4 =

Addition Fact Linking #9 Name _____

Think Page: Doubles +2 In the "think cloud" write the linking doubles fact for each problem (example: the linking doubles fact for $\overset{6}{+4}$ is $\overset{4}{+4}$). Then answer the problem.

4 +6 10	5 +7	5 +3	2 +4	7 +9
3 +5	6 +4	9 +7	5 +7	6 +8
4 +2	2 +4	8 +6	9 +7	6 +4
5 +3	7 +5	7 +9	3 +5	6 +8
8 +6	4 +2	7 +5	4 +6	9 +7

Addition Fact Linking #10 Name _____

Mixed Fact Group Practice Page: Doubles, Doubles +1, and Doubles +2 Answer as many as possible. Look for linking strategies. For example, for 6+8, think 6+6 plus 2.

6 +6	6+7 =	7+8 =	8+8 =
7+9 =	5+5 =	8+6 =	9+8 =
6+4 =	8 +7	4 +2	5+7 =
6+8 =	3 +3	4+3 =	5 +6
5+4 =	7 +7	9+7 =	8+7 =
10+10 =	6+5 =	4 +5	5 +7
2+4 =	3+5 =	9 +8	7+5 =
7 +9	9 +9	7 +6	4 +6
3 +4	8 +6	9+9 =	5 +3
8 +8	7 +5	6 +7	8 +7
4+4 =	9 +7	6 +8	8+9 =

Addition Fact Linking #11 Name _____

Practice Page: Doubles +2 Link to base facts: doubles.

7+9 =	8+6 =	9 +7	5+7 =
5 +7	6+8 =	8 +6	6+4 =
7+5 =	9+7 =	4+6 =	7 +9
5 +7	4 +6	7+5 =	8+6 =
9+7 =	6+8 =	2+4 =	7+9 =
6 +8	6 +4	5 +7	4 +6
7 +9	5+3 =	9 +7	7 +9
4+6 =	7 +5	6 +4	6 +8
7+9 =	5+7 =	8 +6	9+7 =
6+4 =	6 +8	7 +5	4+6 =

Addition Fact Linking #12 Name _____

One-Minute Progress Check: Doubles +2 Complete as many of these problems as possible in 1 minute.

4+6 =	8+6 =	9 +7	6 +8
8 +6	5 +7	6+8 =	6+4 =
7+9 =	4 +6	6 +4	7 +9
7 +5	4 +2	7+5 =	5 +7
9+7 =	5 +7	7+9 =	3+5 =
6+8 =	6+4 =	7 +5	4+6 =
5+7 =	8 +6	9+7 =	3+5 =
7 +9	6+8 =	4+6 =	6 +8
4 +6	7+5 =	8+6 =	7 +9
9 +7	5+7 =	6 +8	6 +4

Addition Fact Linking #13 Name _____

Base Facts Practice Page: +1 and +0 (0-20) Answer as many as possible.

6+0 =	11 +1	7+0 =	20 +1
7 +1	1+3 =	1 +20	1+11 =
0 +4	17 +1	1+1 =	2 +0
6 +1	13+0 =	1 +14	1+8 =
16+0 =	8 +1	12+0 =	9+0 =
19 +1	12+0 =	0+16 =	13 +0
15+0 =	1 +18	3+0 =	0 +5
10 +1	20 +0	7+1 =	2+1 =
9+1 =	0+19 =	13 +0	13+1 =
1 +16	20 +1	15+0 =	0 +10

Addition Fact Linking #14 Name _____

Base Facts Practice Page: +1 and +0 (0-100) Answer as many as possible.

29+1 =	62+0 =	66 +1	79 +1
22+1 =	1 +77	49 +1	0+44 =
69 +1	89+1 =	0+30 =	20 +1
26+0 =	44 +0	69 +1	1+14 =
1 +99	29 +1	1+55 =	77 +1
39 +1	47+1 =	63 +1	0+28 =
35+0 =	0 +97	44+0 =	22 +1
59+1 =	53+0 =	35 +0	31+1 =
71 +0	1+59 =	62+0 =	56 +0
48+1 =	44 +1	0+80 =	33 +0

Addition Fact Linking #15 Name _____

Base Facts Practice Page: +1 and +0 (0-1000) Answer as many as possible.

199+1 =	765 +0	1+121 =	699 +1
876+1 =	0 +543	888 +1	210+0 =
799+1 =	499+1 =	923 +0	1 +82
1 +899	999+1 =	19+1 =	444 +0
321 +0	987 +0	543+0 =	399+1 =
299+1 =	699 +1	119 +1	432+0 =
119+0 =	416 +0	402 +1	654 +0
210 +0	1+381 =	628+0 =	164+0 =
599+1 =	326+0 =	706 +1	1 +177
432 +0	234 +1	1+548 =	830+0 =

Addition Fact Linking #16 Name _____

One-Minute Progress Check: +1 and +0 Complete as many of these problems as possible in 1 minute.

199+1 =	765+0 =	1+359 =	1+19 =
876+1 =	0+543 =	1+54 =	0+210 =
799+1 =	183+1 =	77+0 =	1+99 =
1+899 =	359+1 =	21+1 =	606+0 =
0+321 =	0+604 =	555+0 =	1+799 =
299+1 =	1+21 =	0+765 =	0+539 =
119+0 =	0+490 =	899+1 =	66+0 =
210+0 =	19+1 =	302+0 =	490+0 =
1+199 =	543+0 =	249+1 =	1+13 =
0+432 =	88+1 =	1+668 =	89+0 =

Addition Fact Linking #17 Name _____

Base Facts Practice Page: +10 (0-9) Answer as many as possible.

10+3 =	0+10 =	10 +7	6+10 =
4+10 =	8 +10	10+8 =	6 +10
9 +10	10 + 7	10+4 =	3+10 =
10+6 =	3 +10	10+5 =	7 +10
9 +10	5 +10	10+9 =	7+10 =
10 +0	10+8 =	10+3 =	4 +10
9+10 =	10 +4	10 +9	10 +3
8 +10	10+0 =	10 +6	10 +8
3+10 =	1+10 =	0 +10	8+10 =
10 +5	10+7 =	5+10 =	10 +6

Addition Fact Linking #18 Name _____

Base Facts Practice Page: +10 (0-100) Answer as many as possible.

16+10 =	54+10 =	66+10 =	86+10 =
22+10 =	10+66 =	10+47 =	10+16 =
19+10 =	10+91 =	10+46 =	4+10 =
10+36 =	47+10 =	10+54 =	2+10 =
49+10 =	10+64 =	10+18 =	7+10 =
10+86 =	10+55 =	10+19 =	34+10 =
17+10 =	10+4 =	10+79 =	62+10 =
10+84 =	60+10 =	10+24 =	18+10 =
79+10 =	98+10 =	52+10 =	11+10 =
91+10 =	10+62 =	10+87 =	55+10 =

Addition Fact Linking #19 Name _____

One-Minute Progress Check: +10 Complete as many of these problems as possible in 1 minute.

6+10 =	54+10 =	10+85 =	10+9 =
16+10 =	10+66 =	44+10 =	10+22 =
9+10 =	10+51 =	10+6 =	34+10 =
10+16 =	10+22 =	66+10 =	92+10 =
79+10 =	10+44 =	85+10 =	67+10 =
10+27 =	10+25 =	10+44 =	84+10 =
47+10 =	10+92 =	10+59 =	63+10 =
10+14 =	6+10 =	10+74 =	10+88 =
43+10 =	48+10 =	51+10 =	91+10 =
88+10 =	10+20 =	10+77 =	10+79 =

Addition Fact Linking #20 Name _____

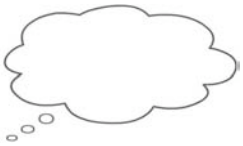
Think Page: +9 In the "think cloud" write the linking +10 fact for each problem (example: the linking +10 fact for 4+9 is 4+10). Then answer the problem.

4+9 = 13 (4+10)	7+9 =	1+9 =	9+3 =
5+9 =	9+4 =	3+9 =	8+9 =
9+4 =	4+9 =	6+9 =	9+9 =
6+9 =	1+9 =	9+6 =	2+9 =
7+9 =	9+5 =	8+9 =	9+7 =
9+2 =	9+3 =	7+9 =	8+9 =
3+9 =	9+4 =	5+9 =	9+9 =

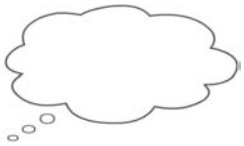
Addition Fact Linking #21 Name _____

Think Page: +9 In the "think cloud" write the linking +10 fact for each problem (example: the linking +10 fact for 42+9 is 42+10). Then answer the problem.

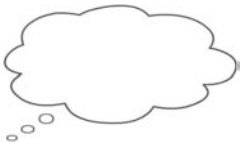
42+9 = 51



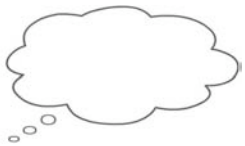
27+9 =



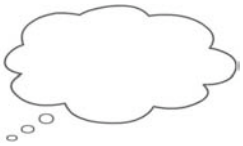
71+9 =



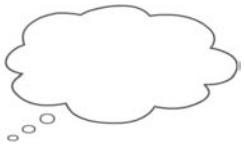
9+13 =



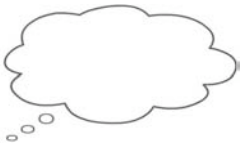
54+9 =



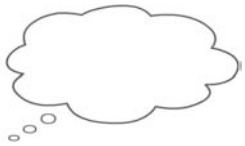
9+47 =



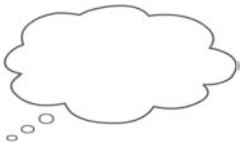
63+9 =



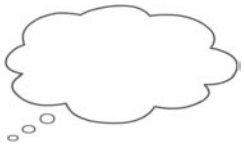
18+9 =



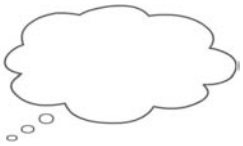
9+4 =



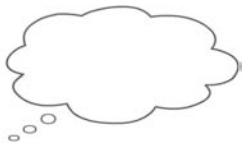
45+9 =



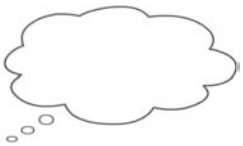
36+9 =



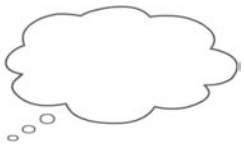
9+89 =



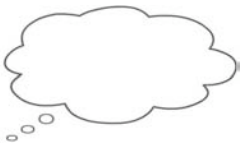
65+9 =



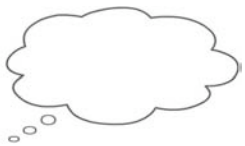
14+9 =



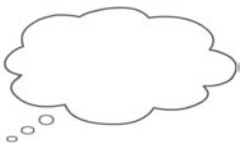
9+96 =



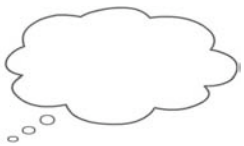
23+9 =



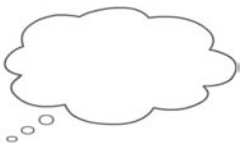
77+9 =



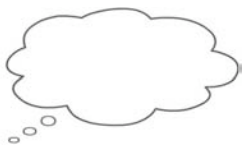
9+5 =



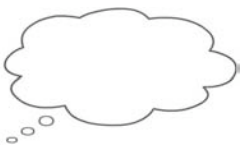
84+9 =



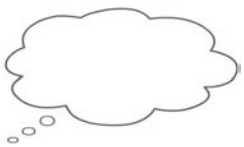
9+17 =



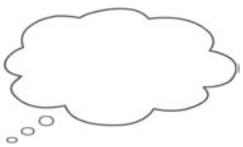
9+82 =



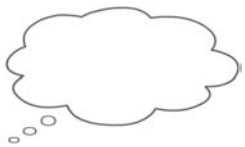
9+31 =



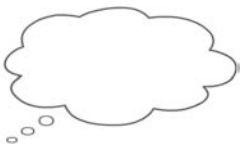
73+9 =



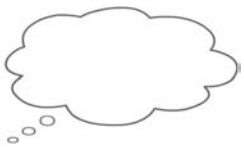
48+9 =



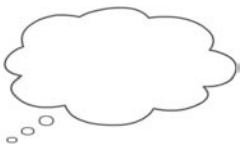
3+9 =



9+44 =



55+9 =



96+9 =

Addition Fact Linking #22 Name _____

Mixed Fact Group Practice Page: +10 and +9 Answer as many as possible. Look for linking strategies. For example, for 4+9, think 4+10 minus 1.

6 +10	4+9 =	10 +4	6+10 =
2+10 =	9 +7	10 +7	10+2 =
9+2 =	10 +1	9+1 =	4 +10
10+6 =	10 +7	9 +7	8+9 =
6 +9	4+10 =	9 +4	7+10 =
10 +17	17+9 =	13 +10	13+9 =
11+10 =	11 +9	14 +9	14+10 =
10+34 =	34 +9	10+24 =	24 +9
9 +18	18+10 =	31 +9	31+10 =
15 +10	15+9 =	10+57 =	9 +57

Addition Fact Linking #23 Name _____

Practice Page: +9 (0-9) Link to base facts: +10 (think +10 minus1).

9+3 =	0+9 =	9 +7	6 +9
4+9 =	6+9 =	9+8 =	9 +6
9+9 =	9 +1	9 +4	4+9 =
8 +9	3 +9	9+5 =	7 +9
7+9 =	5 +9	9 +9	7+9 =
6 +9	7 +9	9+3 =	4+9 =
9+8 =	9+4 =	6 +9	9 +3
4 +9	9 +0	9+6 =	9 +8
3+9 =	1+9 =	8+9 =	3 +9
9 +5	9+7 =	5+9 =	9 +7

Addition Fact Linking #24 Name _____

Practice Page: +9 (0-100) Link to base facts: +10 (think +10 minus1).

16 +9	54+9 =	9+95 =	86 +9
22 +9	9+66 =	9 +47	9+92 =
19+9 =	9 +91	9+46 =	4 +9
9+36 =	73 +9	9 +57	2+9 =
49 +9	9+64 =	9+18 =	7 +9
9 +47	9+55 =	73 +9	34 +9
17+9 =	61 +9	9+79 =	33+9 =
9+84 =	9 +60	9+24 =	9 +18
93 +9	98+9 =	52 +9	11+9 =
15 +9	9 +62	9+87 =	9+57 =

Addition Fact Linking #25 Name _____

Base Facts Practice Page: The Last Ten Addition Facts – Part 1 (3+7 and 4+7) and Part 2 (2+8 and 3+8) Answer as many as possible.

.

3+7 =	4 +7	7+3 =	3 +7
7 +4	4+7 =	3+7 =	4 +7
7+4 =	7 +3	7 +4	7+3 =
4 +7	7+3 =	3+7 =	4+7 =
3 +7	7+4 =	7 +4	7 +3
8+2 =	3 +8	8 +3	8+3 =
2+8 =	8+2 =	3+8 =	2 +8
8 +2	3+8 =	8 +3	8+4 =
3 +8	8+3 =	2+8 =	8 +2
2+8 =	2 +8	3 +8	8 +3

Addition Fact Linking #26 Name _____

Mixed Facts Practice Page: The Last Ten Addition Facts – Part 1 (3+7 and 4+7) and Part 2 (2+8 and 3+8) Answer as many as possible.

.

8+2 =	7 +4	7+3 =	8 +2
7 +4	8+4 =	3+7 =	4 +7
2+8 =	7 +3	2 +8	7+3 =
4 +7	7+3 =	3+8 =	4+7 =
3 +7	7+4 =	4 +7	7 +3
7 +4	3 +8	8 +3	8+3 =
2+8 =	3+7 =	3+8 =	2 +8
3 +7	3+8 =	8 +3	4+7 =
3 +8	8+3 =	7+4 =	8 +2
3+7 =	8+2 =	3 +8	8 +3

Addition Fact Linking #27 Name _____

Base Facts Practice Page: The Last Ten Addition Facts – Part 3 (4+8 and 5+8) and Part 4 (2+5, 2+6, 2+7, and 3+6) Answer as many as possible.

.

4+8 =	5 +8	8+4 =	4 +8
8 +5	5+8 =	4+8 =	5 +8
8+5 =	8 +4	8 +5	8+4 =
5 +8	8+4 =	4+8 =	5+8 =
4 +8	8+5 =	8 +5	8 +4
5+2 =	3 +6	7 +2	6+3 =
2+6 =	3+6 =	2+7 =	2 +7
6 +2	2+5 =	6 +3	6+2 =
3 +6	7+2 =	2+5 =	6 +2
2+7 =	2 +5	2 +6	8 +3

Addition Fact Linking #28 Name _____

Mixed Facts Practice Page: The Last Ten Addition Facts – All parts Answer as many as possible.

.

3+7 =	2 +6	7+2 =	3 +7
8 +2	4+7 =	3+6 =	4 +8
6+2 =	5 +2	7 +4	6+3 =
4 +7	7+3 =	2+5 =	8+4 =
2 +7	7+4 =	6 +2	7 +2
8+2 =	3 +8	5 +8	2+6 =
8+5 =	5+2 =	3+8 =	2 +8
6 +3	5+8 =	2 +5	8+4 =
8 +4	8+3 =	2+8 =	7 +3
2+7 =	8 +5	3 +6	8 +3

Addition Fact Linking #29 Name _____

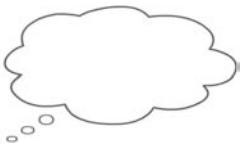
One-Minute Progress Check: The Last Ten Addition Facts Complete as many of these problems as possible in 1 minute.

8+5 =	8 +2	5+2 =	6+2 =
3 +6	4+8 =	7 +3	2+6 =
4+7 =	3 +7	2 +8	4 +8
5 +2	6+3 =	2+5 =	3+6 =
8 +4	7+2 =	7 +4	7 +2
2+8 =	6 +3	8+3 =	8+4 =
2 +6	4 +7	3+7 =	5 +8
2 +7	3 +8	8 +3	3+8 =
8 +5	7+4 =	5+8 =	2 +5
6 +2	2+7 =	8+2 =	7+3 =

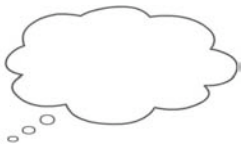
Subtraction Fact Linking #1 Name _____

Think Page: Subtraction Doubles In the "think cloud" write the linking addition doubles fact for each problem (example: the linking doubles fact for 4-2 is 2+2). Then answer the problem.

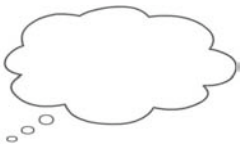
4-2 = 2



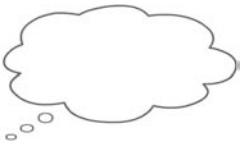
10-5 =



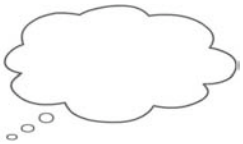
16-8 =



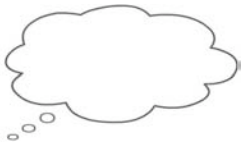
8-4 =



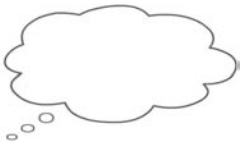
6-3 =



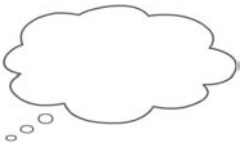
12-6 =



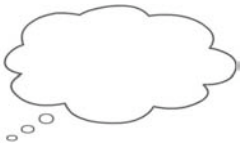
18-9 =



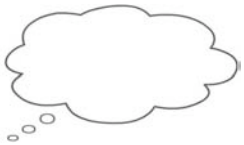
14-7 =



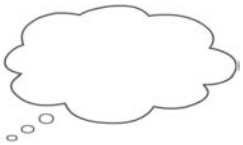
20-10 =



4+3 =



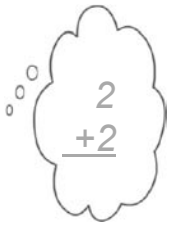
14-7 =



16-8 =

In the "think cloud" write the linking addition doubles fact for each problem (example: the linking doubles fact for $\overset{4}{\underline{-2}}$ is $\overset{2}{\underline{+2}}$). Then answer the problem.

4
-2
2



14
-7

6
-3

18
-9

16
-8

10
-5

8
-4

12
-6

20
-10

14
-7

Subtraction Fact Linking #2 Name _____

Mixed Fact Group Practice Page: Addition Doubles and Subtraction Doubles

Answer as many as possible. For subtraction, think addition (for example, for 8-4=❑ think 4+❑=8).

2+2 =	6+6 =	20 -10	16-8 =
10 - 5	3 +3	8-4 =	2 +2
4+4 =	18-9 =	8+8 =	10 +10
10+10 =	4 - 2	12-6 =	5 +5
7+7 =	16 - 8	6 - 3	4 +4
14 - 7	8 +8	5+5 =	7 +7
3+3 =	18 -9	6 +6	4-2 =
8 - 4	10-5 =	9+9 =	16-8 =
18 - 9	14 -7	12 - 6	6-3 =
12-6 =	20-10 =	14-7 =	9 +9

Subtraction Fact Linking #3 Name _____

Practice Page: Subtraction Doubles Link to base facts: addition doubles. (Remember, 8-4=❑ links to 4+❑=8.)

14-7 =	10-5 =	16 - 8	6 - 3
20 -10	4 - 2	20-10 =	12-6 =
14 - 7	8-4 =	10 - 5	18 - 9
6 - 3	12 - 6	16-8 =	4-2 =
4 - 2	8-4 =	18-9 =	14 -7
20 -10	18 - 9	4-2 =	6-3 =
14-7 =	16-8 =	12 - 6	16-8 =
18 - 9	8 - 4	20-10 =	18-9 =
6-3 =	16 - 8	14 - 7	10 - 5
8-4 =	10-5 =	8 - 4	12-6 =

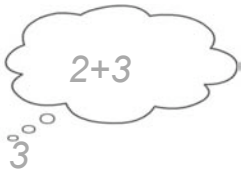
Subtraction Fact Linking #4 Name _____

One-Minute Progress Check: Subtraction Doubles Complete as many of these problems as possible in 1 minute.

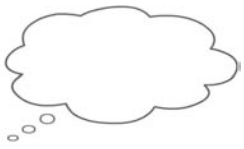
12-6 =	10 - 5	20 - 10	18 - 9
4 - 2	16-8 =	8-4 =	4-2 =
8 - 4	12 - 6	6 - 3	20-10 =
10-5 =	18-9 =	4-2 =	10 - 5
14-7 =	16 - 8	6-3 =	8-4 =
18-9 =	14 - 7	10-5 =	16 - 8
6-3 =	12-6 =	14-7 =	12 - 6
14-7 =	16 - 8	18 - 9	16-8 =
20 - 10	12 - 6	6 - 3	8 - 4
4 - 2	20-10 =	14 - 7	18-9 =

Subtraction Fact Linking #5 Name _____

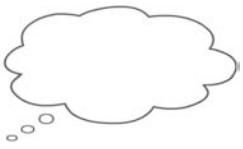
Think Page: Subtraction Doubles +1 In the "think cloud" write the linking doubles +1 fact for each problem (example: 5-2=❑ links to 2+❑=5, so the linking doubles +1 fact for 5-2 is 2+3). Then answer the problem.



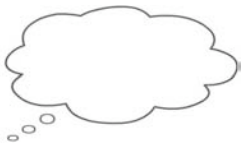
5-2 = 3



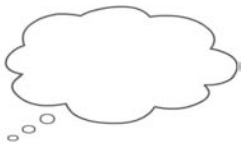
11-5 =



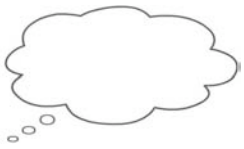
5-3 =



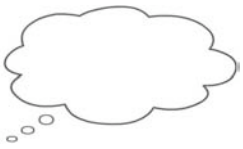
7-4 =



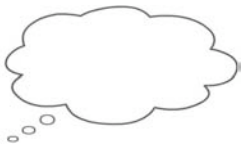
17-8 =



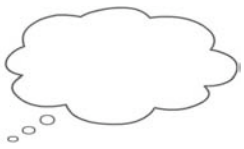
15-8 =



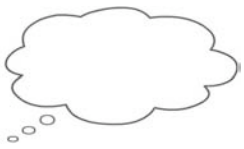
15-7 =



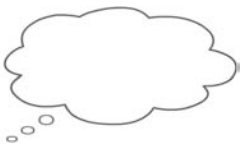
11-6 =



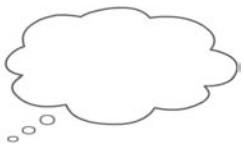
7-4 =



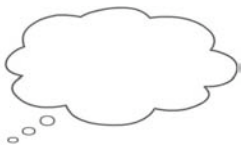
7-3 =



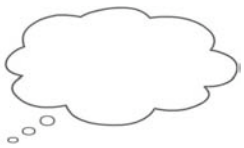
13-6 =



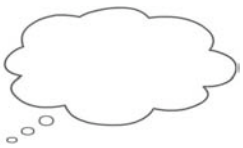
17-9 =



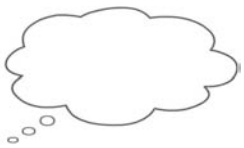
13-7 =



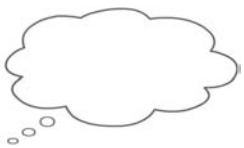
9-5 =



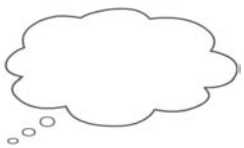
11-6 =



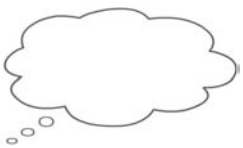
9-4 =



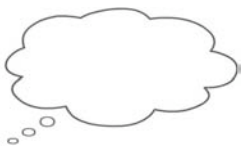
15-8 =



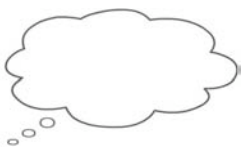
13-7 =



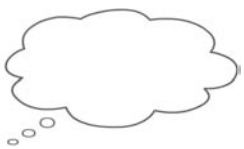
9-5 =



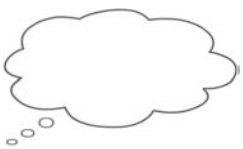
17-8 =



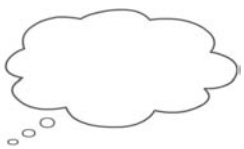
13-6 =



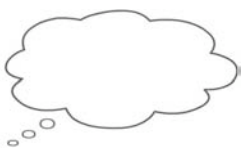
11-5 =



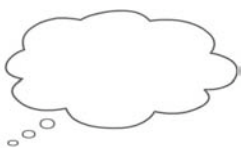
5-2 =



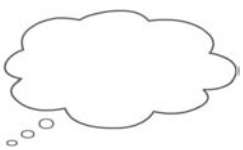
15-7 =



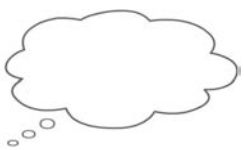
7-3 =



5-3 =



17-9 =



9-4 =

Subtraction Fact Linking #6 Name _____

Think Page: Subtraction Doubles +1 In the "think cloud" write the linking doubles +1 fact for each problem (example: 9-4 = ❑ links to 4+❑ = 9, so the linking doubles +1 fact for 9-4 is 4+5). Then answer the problem.

11 -6 = 5 (6 +5)	15 -7 =	7 -3 =	5 -3 =	17 -9 =
9 -5 =	11 -5 =	13 -6 =	9 -4 =	15 -8 =
5 -2 =	7 -4 =	15 -7 =	17 -8 =	13 -7 =
11 -6 =	13 -6 =	17 -9 =	11 -5 =	17 -8 =
13 -7 =	9 -5 =	15 -8 =	9 -4 =	7 -3 =

Subtraction Fact Linking #7 Name _____

Mixed Fact Group Practice Page: Subtraction and Addition Doubles +1 Answer as many as possible. For subtraction, think addition (for example, for 13-7=❑ think 7+❑=13).

6 +7	5 - 3	7+8 =	15-8 =
5 +6	11-5 =	17 - 9	9+8 =
9-4 =	8 +7	7-3 =	11 - 6
11-6 =	2+3 =	4+3 =	15 - 8
7 - 3	13 - 7	5+4 =	15 - 7
13-6 =	6+5 =	13 - 6	10 +9
7-4 =	4+5 =	9 - 5	5+6 =
8+7 =	11 - 5	7+6 =	17-8 =
5-2 =	3 +2	7 - 4	8 +9
17 - 8	3+4 =	5 -2	8+7 =

Subtraction Fact Linking #8 Name _____

Practice Page: Subtraction Doubles +1 Link to base facts: addition doubles +1 (remember, 9-4=❑ links to 4+❑=9).

13-7 =	11-5 =	15-8 =	7 - 3
15 - 8	5 - 2	17 - 8	13 - 6
15-7 =	7-4 =	11 - 5	17-9 =
5 - 3	11-6 =	17-8 =	5-2 =
9-5 =	7 - 4	13-6 =	17 - 8
17 - 9	5-3 =	9 - 5	11-6 =
13 - 7	15-8 =	11 - 5	17-8 =
13-7 =	7-3 =	13 - 6	13 - 7
7 - 4	17 - 9	15 - 7	5-2 =
9-4 =	11 - 6	9 - 4	7-3 =

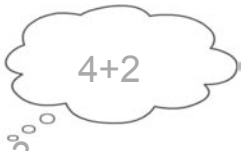
Subtraction Fact Linking #9 Name _____

One-Minute Progress Check: Subtraction doubles +1 Complete as many of these problems as possible in 1 minute.

5 - 2	5-2 =	15 - 8	7 - 3
15-7 =	17 - 9	17-8 =	13-6 =
7 - 3	5-3 =	11-5 =	9-4 =
5 - 3	9-5 =	17 - 8	11 - 6
7-4 =	11-5 =	15-7 =	9 - 5
9 - 5	15 - 8	13 - 7	15-8 =
13-7 =	7 - 4	11 - 5	17-8 =
11-6 =	17-9 =	13 - 7	9 - 4
9-4 =	17 - 9	13-7 =	5 - 3
15 - 7	7-4 =	13 - 6	7-3 =

Subtraction Fact Linking #10 Name _____

Think Page: Subtraction Doubles +2 In the "think cloud" write the linking doubles +2 fact for each problem (example: 10-4=❑ links to 4+❑=10, so the linking doubles +2 fact for 10-4 is 4+6). Then answer the problem.



6-4 = 2

12-5 =

16-9 =

8-3 =

10-6 =

16-7 =

8-5 =

14-6 =

10-4 =

6-2 =

14-8 =

12-7 =

12-7 =

10-6 =

14-6 =

8-5 =

16-9 =

12-5 =

10-6 =

16-7 =

6-2 =

8-3 =

16-9 =

14-8 =

8-5 =

6-4 =

12-7 =

10-4 =

Subtraction Fact Linking #11 Name _____

Think Page: Subtraction Doubles +2 In the "think cloud" write the linking doubles +2 fact for each problem (example: 10-4=❑ links to 4+❑=10, so the linking doubles +2 fact for 10-4 is 4+6). Then answer the problem.

14 -6 = 8 (6 +8)	12 -7 =	8 -3 =	8 -5 =	16 -9 =
8 -5 =	6 -2 =	10 -6 =	6 -4 =	14 -8 =
6 -2 =	10 -4 =	12 -7 =	16 -9 =	16 -7 =
14 -6 =	10 -6 =	16 -9 =	10 -4 =	12 -5 =
16 -7 =	12 -5 =	14 -8 =	6 -4 =	8 -3 =

Subtraction Fact Linking #12 Name _____

Mixed Fact Group Practice Page: Addition and Subtraction Doubles +2 Answer as many as possible. For subtraction, think addition (for example, for 14-6=❑ think 6+❑=14).

14 - 6	14-8 =	8 - 3	16-7 =
9+7 =	5+7 =	4 +6	10-6 =
16 - 9	7 +5	3+5 =	4 +2
6+8 =	2+4 =	8-3 =	6 - 4
10 - 4	12 - 7	6 - 2	16 - 7
14-6 =	12-5 =	10-4 =	5 +7
8-5 =	6-2 =	12 - 5	3+5 =
7 +9	16 - 9	14 - 6	6+4 =
6-4 =	8 +6	16-9 =	5 +3
14 - 8	8 - 5	12-7 =	10 - 6

Subtraction Fact Linking #13 Name _____

Practice Page: Subtraction Doubles +2 Link to base facts: addition doubles +2.
(Remember, 14-6=❑ links to 6+❑=14.)

6-4 =	14 - 6	16 - 7	6-2 =
12 - 7	8 - 5	14-8 =	10-4 =
12-5 =	16-9 =	10 - 4	8 - 3
10-6 =	6 - 4	12 - 5	14-6 =
16-7 =	14 - 8	8-3 =	16 - 9
14-8 =	12 - 7	6 - 2	10 - 6
8 - 5	6-4 =	16-7 =	8-5 =
14-6 =	12 - 5	10 - 6	14 - 8
6 - 2	12-7 =	14 - 6	8 - 3
10-4 =	8-3 =	16 - 9	12-7 =

Subtraction Fact Linking #14 Name _____

One-Minute Progress Check: Subtraction Doubles +2 Complete as many of these problems as possible in 1 minute.

8 - 3	14-6 =	16 - 7	14-8 =
14 - 6	8-3 =	12 - 7	10 - 4
16-9 =	10 - 6	14 - 8	6-4 =
10-6 =	6 - 4	12-5 =	12 - 7
16-7 =	6-2 =	16 - 9	12 - 5
14-8 =	10-4 =	8-5 =	10-6 =
8 - 5	6 - 2	16-7 =	8-5 =
8 - 3	16 - 9	10 - 6	14 - 8
10-4 =	12-5 =	14-6 =	8 - 3
6-2 =	10 - 4	12-7 =	6 - 4

Subtraction Fact Linking #15 Name _____

Base Facts Practice Page: -1 and -0 (0-24) Answer as many as possible.

6-0 =	11 - 1	7-0 =	20 - 1
7 - 1	3-1 =	22 - 1	11-1 =
4 - 0	17 - 1	1+1 =	2 - 0
6 - 1	13-0 =	14 - 1	8-1 =
16-0 =	8 - 1	12-0 =	9 - 0
1 - 0	12-0 =	16-0 =	13 - 0
15-0 =	18 - 1	3-0 =	5 - 0
10 - 1	21 - 0	7-1 =	2-1 =
9-1 =	24-0 =	13 - 0	13 - 1
16-1 =	20-1 =	15-0 =	10 - 0

Subtraction Fact Linking #16 Name _____

Base Facts Practice Page: -1 and -0 (0-100) Answer as many as possible.

29-1 =	62-0 =	66-1 =	79-1 =
22-1 =	77-1 =	49-1 =	44-0 =
69-1 =	89-1 =	30-0 =	20-1 =
26-0 =	44-0 =	69-1 =	14-1 =
99-1 =	29-1 =	55-1 =	77-1 =
39-1 =	47-1 =	63-1 =	28-0 =
35-1 =	97-0 =	44-0 =	22-1 =
55-1 =	53-0 =	35-0 =	31-1 =
71-0 =	59-1 =	62-0 =	56-0 =
48-1 =	24-1 =	80-0 =	33-0 =

Subtraction Fact Linking #17 Name _____

Base Facts Practice Page: -1 and -0 (0-1000) Answer as many as possible.

199-1 =	765-0 =	121-1 =	700-1 =
876-1 =	543-0 =	889-1 =	210-0 =
799-1 =	500-1 =	923-0 =	82-1 =
900-1 =	1,000-1 =	20-1 =	444-0 =
321-0 =	987-0 =	543-0 =	400-1 =
300-1 =	699-1 =	120-1 =	432-0 =
119-0 =	416-0 =	402-1 =	654-0 =
210-0 =	381-1 =	628-0 =	164-0 =
599-1 =	326-0 =	706-1 =	177-1 =
432-0 =	234-1 =	548-1 =	830-0 =

Subtraction Fact Linking #18 Name _____

One-Minute Progress Check: -1 and -0 Complete as many of these problems as possible in 1 minute.

200-1 =	765-0 =	708-1 =	20-1 =
876-1 =	543-0 =	54-1 =	210-0 =
800-1 =	183-1 =	77-0 =	100-1 =
900-1 =	360-1 =	230-1 =	606-0 =
320-1 =	604-0 =	555-0 =	316-0 =
300-1 =	21-1 =	475-0 =	539-0 =
119-0 =	490-0 =	861-1 =	66-0 =
210-0 =	727-1 =	302-0 =	903-0 =
600-1 =	554-0 =	250-1 =	13-1 =
432-0 =	88-1 =	668-1 =	89-0 =

Subtraction Fact Linking #19 Name _____

Base Facts Practice Page: -10 (10-100) Answer as many as possible.

16-10 =	54-10 =	95-10 =	86-10 =
22-10 =	66-10 =	45-10 =	92-10 =
19-10 =	91-10 =	46-10 =	14-0 =
36-10 =	73-10 =	57-10 =	12-10 =
49-10 =	64-10 =	18-10 =	17-10 =
47-10 =	55-10 =	73-10 =	34-10 =
17-10 =	61-10 =	79-10 =	33-10 =
84-10 =	60-10 =	24-10 =	18-10 =
93-10 =	98-10 =	52-10 =	11-10 =
15-10 =	62-10 =	87-10 =	57-10 =

Subtraction Fact Linking #20 Name _____

One-Minute Progress Check: -10 Complete as many of these problems as possible in 1 minute.

16-10 =	54-10 =	85-10 =	46-10 =
12-10 =	66-10 =	37-10 =	22-10 =
19-10 =	51-10 =	16-10 =	34-10 =
16-10 =	33-10 =	77-10 =	92-10 =
79-10 =	44-10 =	18-10 =	67-10 =
27-10 =	25-10 =	93-10 =	84-10 =
47-10 =	81-10 =	59-10 =	63-10 =
14-10 =	71-10 =	74-10 =	88-10 =
43-10 =	48-10 =	22-10 =	91-10 =
15-10 =	20-10 =	77-10 =	47-10 =

Subtraction Fact Linking #21 Name _____

Think Page: -9 In the "think cloud" write the linking -10 fact for each problem (example: the linking -10 fact for 14-9 is 14-10). Then answer the problem.

42-9 = 33

27-9 =

71-9 =

13-9 =

54-9 =

47-9 =

63-9 =

18-9 =

14-9 =

45-9 =

36-9 =

89-9 =

65-9 =

14-9 =

96-9 =

23-9 =

77-9 =

15-9 =

84-9 =

17-9 =

82-9 =

31-9 =

73-9 =

48-9 =

13-9 =

44-9 =

55-9 =

96-9 =

Subtraction Fact Linking #22 Name _____

Think Page: -9 In the "think cloud" write the linking -10 fact for each problem (example: the linking -10 fact for 14-9 is 14-10). Then answer the problem.

14 -9 = 5 (14 - 10)	92 -9	88 -9	85 -9	57 -9
23 -9	71 -9	20 -9	61 -9	89 -9
46 -9	60 -9	32 -9	33 -9	47 -9
54 -9	19 -9	76 -9	17 -9	72 -9
36 -9	82 -9	24 -9	65 -9	99 -9

Subtraction Fact Linking #23 Name _____

Mixed Fact Group Practice Page: -10 and -9 Answer as many as possible. Look for linking strategies. For example, for 14-9, think 14-10 plus 1.

16 - 10	14-9 =	14 - 10	16-10 =
12-10 =	17 - 9	17-10 =	10 - 10
12-9 =	11 - 10	11 - 9	14-10 =
16-10 =	17 - 10	17 - 9	18-9 =
16-9 =	14-10 =	14-9 =	18 - 10
17 - 10	17 - 9	13 - 10	13 - 9
11 - 10	11 - 9	14-9 =	14-10 =
34-10 =	34 - 9	24-10 =	24 - 9
18-9 =	18-10 =	31 - 9	31-10 =
15 - 10	15 - 9	57-10 =	57-9 =

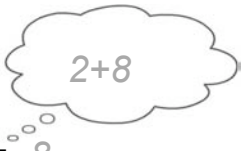
Subtraction Fact Linking #24 Name _____

Practice Page: -9 (10-100) Link to base facts: -10 (think -10 +1).

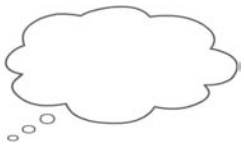
16 - 9	54-9 =	95-9 =	86 - 9
22-9 =	66-9 =	47 - 9	92 - 9
19-9 =	91 - 9	46 - 9	14-9 =
36-9 =	73 - 9	57 - 9	12-9 =
49 - 9	64 - 9	18-9 =	17-9 =
47 - 9	55-9 =	73 - 9	34-9 =
17 - 9	61-9 =	79-9 =	33 - 9
84-9 =	60 - 9	24-9 =	18 - 9
93-9 =	98 - 9	52 - 9	11-9 =
15 - 9	62-9 =	87 - 9	57-9 =

Subtraction Fact Linking #25 Name _____

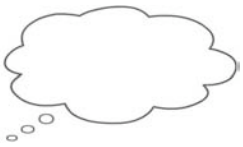
Think Page: Last Ten Subtraction Facts In the "think cloud" write the linking last ten *addition* number fact for each problem (example: the linking addition fact for 13-5 is 5+8). Then answer the problem.



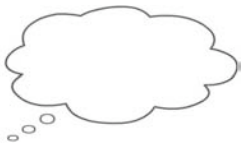
10-2 = 8



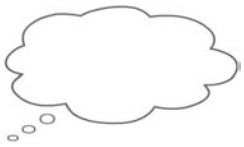
9-7 =



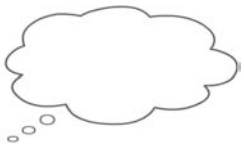
11-3 =



9-6 =



8-6 =



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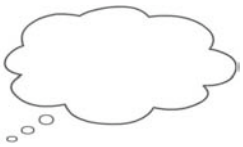

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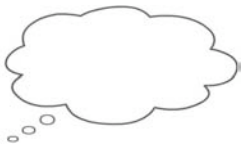
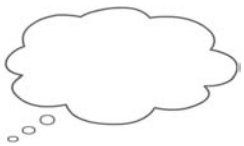
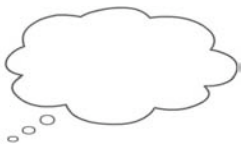


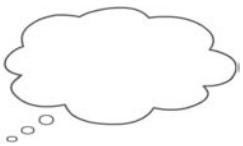
10-3 =



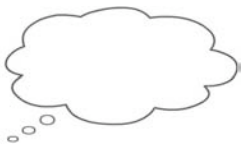
13-8 =



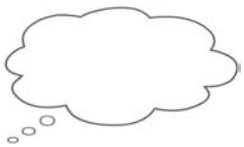
11-4 =



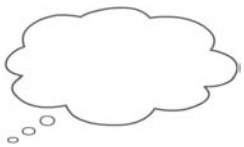
7-2 =



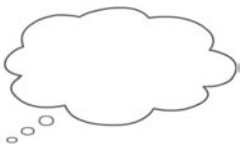
10-8 =



11-3 =



11-7 =



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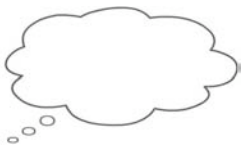
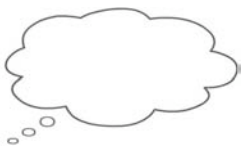
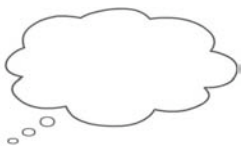


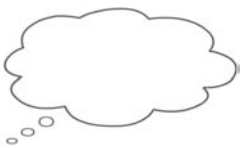
9-3 =



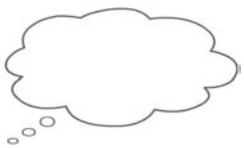
12-8 =



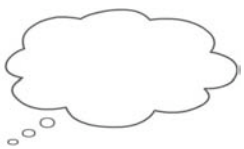
13-5 =



7-5 =



12-4 =



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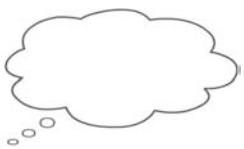
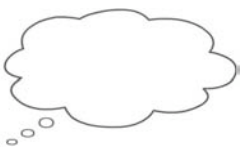
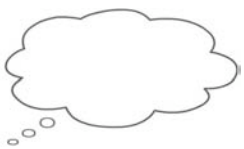


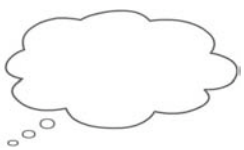
8-2 =



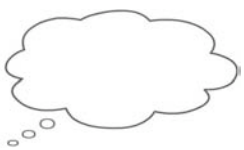
8-6 =



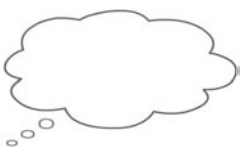
11-4 =



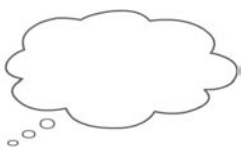
10-7 =



13-8 =



9-6 =



9-2 =

Subtraction Fact Linking #26 Name _____

Think Page: Last Ten Subtraction Facts In the "think cloud" write the linking last ten *addition* number fact for each problem (example: the linking addition fact for 13-5 is 5+8). Then answer the problem.

13 − 5 = 8 (5 + 8)	11 − 4 =	9 − 6 =	8 − 6 =	11 − 8 =
10 − 2 =	9 − 2 =	12 − 8 =	13 − 8 =	7 − 5 =
11 − 4 =	10 − 7 =	12 − 4 =	13 − 8 =	10 − 3 =
9 − 7 =	9 − 2 =	11 − 7 =	8 − 2 =	10 − 8 =
7 − 2 =	11 − 3 =	10 − 7 =	9 − 3 =	12 − 4 =

Subtraction Fact Linking #27 Name _____

Mixed Fact Group Practice Page: Last Ten Addition and Subtraction Facts Answer as many as possible. For subtraction, think addition (for example, for 7-5=❑ think 5+❑=7).

2+5 =	9 - 2	7 - 5	2+8 =
7 +4	12-8 =	3+7 =	11 - 3
7-2 =	11 - 4	10 - 8	3+6 =
5 +2	13-5 =	8+5 =	4+8 =
8 +4	10-2 =	9 - 6	10 - 3
12 - 4	4+7 =	11-7 =	8 +2
8-6 =	6 +2	7 +3	10-7 =
7 +2	3+8 =	8 +3	11-8 =
9-7 =	9-3 =	2+6 =	8 - 2
2+7 =	5 +8	13 - 8	6 +3

Subtraction Fact Linking #28 Name _____

Practice Page: The Last Ten Subtraction Facts Link to base facts: last ten addition number facts. (Remember, 7-5=❑ links to 5+❑=7.)

7-5 =	13 - 5	7 - 2	10 - 8
11 - 7	9-6 =	10 - 7	11 - 4
11-3 =	10 - 3	12 - 8	7 - 5
7-2 =	9-3 =	13-5 =	12-8 =
12 - 4	10-8 =	9 - 6	9 - 2
8-2 =	10-3 =	11-7 =	11-4 =
8 - 6	10-2 =	10-7 =	9-7 =
9-2 =	11-8 =	11 - 3	12-4 =
13-8 =	9 - 3	8-6 =	8 - 2
9 - 7	13 - 8	11 - 8	10 - 2

Subtraction Fact Linking #29 Name _____

One-Minute Progress Check: The Last Ten Subtraction Facts Complete as many of these problems as possible in 1 minute.

13-5 =	10 - 2	7-2 =	8-2 =
9 - 6	12-8 =	10 - 3	8-6 =
11-7 =	9-2 =	12 - 8	12 - 4
7 - 2	9-3 =	7-5 =	9-6 =
12-4 =	10-8 =	11 - 7	9 - 2
8 - 6	9 - 3	11-3 =	12-4 =
11 - 4	10 - 8	7 - 5	13 - 8
9 - 7	11 - 8	11 - 3	11-8 =
13-8 =	11-4 =	13 - 5	9-7 =
8 - 2	10 - 7	10-2 =	10-7 =

Multiplication Fact Linking #1 Name _____

Think Page: ×2 In the "think cloud" write the swap for each problem (example: the swap for 3×2 is 2×3). Then answer the problem.

9×2 = *18*

3×2 =

2×7 =

2×4 =

10×2 =

6×2 =

5×2 =

7×2 =

2×8 =

4×2 =

2×6 =

2×10 =

2×7 =

2×5 =

8×2 =

2×9 =

8×2 =

6×2 =

4×2 =

2×3 =

7×2 =

9×2 =

10×2 =

9×2 =

2×8 =

2×3 =

2×6 =

5×2 =

Multiplication Fact Linking #2 Name _____

Think Page: ×2 In the "think cloud" write the swap for each problem (example: the swap for 3×2 is 2×3). Then answer the problem.

2 ×6 12	6 ×2	8 ×2	4 ×2	2 ×3	2 ×9
5 ×2		6 ×2	7 ×2	5 ×2	2 ×8
3 ×2		2 ×4	8 ×2	9 ×2	2 ×7
2 ×10		7 ×2	2 ×9	6 ×2	9 ×2
2 ×7		2 ×5	2 ×3	10 ×2	4 ×2

Multiplication Fact Linking #3 Name _____

Base Fact Practice Page: ×2 Answer as many as possible. Look for swaps (for example, 4×2 is the same as 2×4).

2×2 =	2×10 =	2 ×3	2×8 =
6 ×2	5 ×2	2 ×9	2 ×4
2×7 =	4 ×2	8 ×2	2 ×10
10×2 =	5×2 =	2×7 =	2 ×5
2×4 =	8 ×2	2×3 =	9 ×2
7 ×2	3 ×2	9 ×2	2×6 =
6×2 =	7×2 =	2 ×6	2 ×2
4×2 =	2 ×4	9×2 =	8×2 =
2×3 =	2×6 =	10 ×2	3×2 =
2 ×8	2×5 =	2 ×7	2×9 =

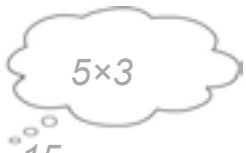
Multiplication Fact Linking #4 Name _____

One-Minute Progress Check: ×2 Complete as many of these problems as possible in 1 minute.

6×2 =	2 ×5	10 ×2	2×9 =
2 ×2	2×8 =	2×4 =	7 ×2
2 ×4	6 ×2	2 ×3	9 ×2
5 ×2	2 ×9	2×7 =	5 ×2
2×7 =	8 ×2	2×4 =	4×2 =
9×2 =	2×3 =	5×2 =	2 ×10
3 ×2	2×6 =	2×2 =	2 ×6
2×5 =	10×2 =	9 ×2	8×2 =
6×2 =	2 ×8	3×2 =	4 ×2
2 ×7	2×10 =	7×2 =	8 ×2

Multiplication Fact Linking #5 Name _____

Think Page: ×5 In the "think cloud" write the swap for each problem (example: the swap for 3×5 is 5×3). Then answer the problem.



3×5 = 15

6×5 =

5×3 =

5×4 =

5×8 =

9×5 =

5×2 =

7×5 =

8×5 =

5×7 =

5×10 =

8×5 =

5×7 =

2×5 =

5×6 =

4×5 =

9×5 =

6×5 =

5×9 =

5×8 =

7×5 =

4×5 =

10×5 =

5×7 =

5×4 =

2×5 =

5×3 =

5×8 =

Multiplication Fact Linking #6 Name _____

Think Page: ×5 In the "think cloud" write the swap for each problem (example: the swap for 3×5 is 5×3). Then answer the problem.

5 ×6 30	6 ×5	8 ×5	4 ×5	5 ×3	5 ×9
5 ×3		6 ×5	7 ×5	5 ×2	5 ×8
3 ×5		5 ×4	8 ×5	9 ×5	5 ×7
5 ×10		7 ×5	5 ×9	6 ×5	9 ×5
5 ×7		2 ×5	5 ×8	10 ×5	4 ×5

Multiplication Fact Linking #7 Name _____

Base Fact Practice Page: ×5 Answer as many as possible. Look for swaps (for example, 6×5 is the same as 5×6).

6 ×5	5 ×7	5×8 =	8 ×5
5×9 =	10×5 =	6×5 =	5×5 =
4 ×5	9 ×5	5×3 =	5 ×8
9×5 =	2×5 =	5×4 =	10 ×5
5 ×3	7 ×5	3×5 =	5×6 =
5 ×6	4×5 =	5×10 =	5 ×9
3×5 =	5×7 =	5 ×5	5 ×2
2 ×5	9 ×5	7×5 =	8×5 =
5 ×10	3 ×5	5 ×6	4×5 =
5 ×8	5 ×4	5×2 =	5×7 =

Multiplication Fact Linking #8 Name _____

One-Minute Progress Check: ×5 Complete as many of these problems as possible in 1 minute.

3 ×5	5×2 =	9×5 =	4 ×5
5 ×7	5 ×9	5×8 =	5×6 =
10×5 =	5×3 =	6 ×5	5 ×5
2 ×5	5 ×4	5×7 =	8×5 =
5×4 =	3×5 =	5 ×6	10 ×5
9 ×5	7×5 =	5 ×2	7 ×5
6×5 =	3×5 =	8 ×5	9×5 =
5 ×10	5×5 =	2×5 =	5×10 =
5 ×4	5 ×3	5×8 =	5 ×6
5 ×8	5×9 =	7 ×5	4×5 =

Multiplication Fact Linking #9 Name _____

Think Page: ×9 In the "think cloud" write the swap for each problem (example: the swap for 3×9 is 9×3). Then answer the problem.

3×9 = 27

7×9 =

2×9 =

9×3 =

4×9 =

9×6 =

9×5 =

8×9 =

9×8 =

9×10 =

6×9 =

9×7 =

9×4 =

9×6 =

8×9 =

9×5 =

5×9 =

9×7 =

4×9 =

9×7 =

9×2 =

9×3 =

7×9 =

9×4 =

3×9 =

9×8 =

10×9 =

6×9 =

Multiplication Fact Linking #10 Name _____

Think Page: ×9 In the "think cloud" write the swap for each problem (example: the swap for 3×9 is 9×3). Then answer the problem.

9 ×6 54	6 ×9	8 ×9	4 ×9	9 ×3	5 ×9
9 ×3		6 ×9	7 ×9	9 ×2	9 ×8
3 ×9		9 ×4	8 ×9	9 ×5	9 ×7
9 ×10		7 ×9	5 ×9	6 ×9	9 ×5
9 ×7		2 ×9	9 ×8	10 ×9	4 ×9

Multiplication Fact Linking #11 Name _____

Base Fact Practice Page: ×9 Answer as many as possible. Look for swaps (for example, 4×9 is the same as 9×4).

9×4 =	9 ×8	9 ×7	9×2 =
5 ×9	9×5 =	6×9 =	9 ×3
2 ×9	10 ×9	9 ×6	4 ×9
4×9 =	9 ×4	9×7 =	8 ×9
9 ×9	6 ×9	5×9 =	3×9 =
9×8 =	7×9 =	9 ×3	9×10 =
3 ×9	9 ×2	10×9 =	9 ×5
8×9 =	7 ×9	9×4 =	9 ×8
2×9 =	6 ×9	9×6 =	7×9 =
9×9 =	9×3 =	9 ×10	4×9 =

Multiplication Fact Linking #12 Name _____

One-Minute Progress Check: ×9 Complete as many of these problems as possible in 1 minute.

3 ×9	9×2 =	7 ×9	4 ×9
9 ×7	9 ×9	9×8 =	9 ×6
9×10 =	9×3 =	6×9 =	9×5 =
2 ×9	5 ×9	9×7 =	8×9 =
4×9 =	3×9 =	9×6 =	10 ×9
9×9 =	7×9 =	9 ×2	9 ×4
9 ×5	9×3 =	8 ×9	9 ×6
9 ×10	5×9 =	2×9 =	10×9 =
9×4 =	9 ×3	6 ×9	3×9 =
9 ×8	8×9 =	7 ×9	4×9 =

Multiplication Fact Linking #13 Name _____

Base Facts Practice Page: ×1 and ×0 (0-10) Answer as many as possible. Look for swaps (for example, 6×0 is the same as 0×6).

6×0 =	8 ×0	0×7 =	1×2 =
0 ×9	1×3 =	1 ×9	1×1 =
4 ×0	1 ×7	4×1 =	2 ×0
6×1 =	3×0 =	1 ×4	0×8 =
0 ×6	8×1 =	0×2 =	9×0 =
1 ×0	10×0 =	0×6 =	3 ×0
0 ×5	1 ×8	0×3 =	0 ×5
10 ×1	0×1 =	7×1 =	2 ×1
9×1 =	0×4 =	7 ×0	3 ×1
1 ×6	1 ×1	5×0 =	0 ×10

Multiplication Fact Linking #14 Name _____

Base Facts Practice Page: ×1 and ×0 (0-100) Answer as many as possible. Look for swaps (for example, 29×1 is the same as 1×29).

29 ×1	62×0 =	66 ×1	79×1 =
22×1 =	1 ×77	49 ×1	0×44 =
69 ×1	89×1 =	0×30 =	20 ×1
26×0 =	44 ×0	69 ×1	1×14 =
1×99 =	29 ×1	1×55 =	77 ×1
39 ×1	47×1 =	63 ×1	0×28 =
35×0 =	0 ×97	44 ×0	22 ×1
59×1 =	53×0 =	35 ×0	31×1 =
71 ×0	1 ×59	62×0 =	56×0 =
48 ×1	44×1 =	0×80 =	33 ×0

Multiplication Fact Linking #15 Name _____

Base Facts Practice Page: ×1 and ×0 (0-1000) Answer as many as possible. Look for swaps (for example, 199×1 is the same as 1×199).

199×1 =	765 ×0	1×121 =	699 ×1
876×1 =	0 ×543	888 ×1	210×0 =
799×1 =	499×1 =	923 ×0	1 ×82
1 ×899	999 ×1	19×1 =	444×0 =
321 ×0	987×0 =	543×0 =	399 ×1
299×1 =	699 ×1	119 ×1	432×0 =
119×0 =	416×0 =	402×1 =	654×0 =
210 ×0	1 ×381	628×0 =	164 ×0
599×1 =	326 ×0	706 ×1	1 ×177
432 ×0	234 ×1	1×548 =	830×0 =

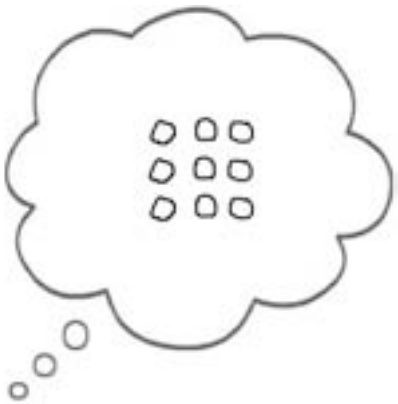
Multiplication Fact Linking #16 Name _____

One-Minute Progress Check: ×1 and ×0 Complete as many of these problems as possible in 1 minute.

199 ×1	765×0 =	1 ×708	19×1 =
876×1 =	543 ×0	54 ×1	0×210 =
799×1 =	1 ×183	77×0 =	99 ×1
1×899 =	359×1 =	231 ×1	606 ×0
0 ×321	604 ×0	555×0 =	1×316 =
299×1 =	1×21 =	475 ×0	539 ×0
119×0 =	490 ×0	861×1 =	66 ×0
0×210 =	727 ×1	302 ×0	903×0 =
599×1 =	554×0 =	249 ×1	1 ×13
432×0 =	88 ×1	1×668 =	89 ×0

Multiplication Fact Linking #17 Name _____

Think Page: Perfect Squares In the "think cloud" draw a picture of the perfect square (see the example). Then answer the problem.



3×3 = *9*

5×5 =

4×4 =

2×2 =

9×9 =

8×8 =

6×6 =

7×7 =

10×10 =

Multiplication Fact Linking #18 Name _____

Base Facts Practice Page: Perfect Squares Answer as many as possible.

6×6 =	5×5 =	10 ×10	9 ×9
4 ×4	8×8 =	4×4 =	8 ×8
7 ×7	6 ×6	3 ×3	10×10 =
4 ×4	9×9 =	7×7 =	5 ×5
7×7 =	8 ×8	3×3 =	4×4 =
9×9 =	3 ×3	5 ×5	1×1 =
6×6 =	7 ×7	3×3 =	6 ×6
5×5 =	10×10 =	9 ×9	8×8 =
10 ×10	6×6 =	3×3 =	4 ×4
5 ×5	8 ×8	7×7 =	9 ×9

Multiplication Fact Linking #19 Name _____

One-Minute Progress Check: Perfect Squares Complete as many of these problems as possible in 1 minute.

2 ×2	6×6 =	10 ×10	8×8 =
5×5 =	3 ×3	4 ×4	2×2 =
4×4 =	9×9 =	8 ×8	10×10 =
10 ×10	2×2 =	6 ×6	5 ×5
7×7 =	8 ×8	3×3 =	4×4 =
9 ×9	1×1 =	5 ×5	1 ×1
3 ×3	7 ×7	6 ×6	2 ×2
7×7 =	5×5 =	9×9 =	8×8 =
1 ×1	6×6 =	10×10 =	3×3 =
4 ×4	1×1 =	7 ×7	9 ×9

Multiplication Fact Linking #20 Name _____

Base Facts Practice Page: The Last Ten Multiplication Facts – Part 1 (3×6, 3×7, and 3×8) and Part 2 (4×6, 4×7, and 4×8) Answer as many as possible.

.

3×7 =	3 ×8	7×3 =	3 ×7
6 ×3	3×8 =	3×6 =	3 ×8
8×3 =	7 ×3	8 ×3	6×3 =
3 ×6	6×3 =	3×7 =	8×3 =
8 ×3	7×3 =	3 ×6	7 ×3
4×7 =	4 ×6	7 ×4	8×4 =
6×4 =	7×4 =	4×8 =	6 ×4
7 ×4	4×8 =	4 ×7	8×4 =
4 ×8	7×4 =	4×6 =	8 ×4
4×6 =	4 ×8	4 ×7	6 ×4

Multiplication Fact Linking #21 Name _____

Mixed Facts Practice Page: The Last Ten Multiplication Facts – Part 1 Part 1 (3×6, 3×7, and 3×8) and Part 2 (4×6, 4×7, and 4×8) Answer as many as possible.

.

4 ×6	7 ×4	7×4 =	3 ×7
6 ×3	3×8 =	4×8 =	8×4 =
8×3 =	7 ×3	7 ×4	6×3 =
7 ×4	3×6 =	3×7 =	8×3 =
8 ×3	7×3 =	4 ×7	7 ×3
4×7 =	3×7 =	3 ×8	8×4 =
6×4 =	7×3 =	4×8 =	6 ×4
3 ×6	6×3 =	4 ×7	3 ×8
4 ×8	7×4 =	4×6 =	8 ×4
4×6 =	4 ×8	3 ×6	6 ×4

Multiplication Fact Linking #22 Name _____

Base Facts Practice Page: The Last Ten Multiplication Facts – Part 3 (6×7 and 6×8) and Part 4 (3×4 and 7×8) Answer as many as possible.

.

6×8 =	6 ×8	7×6 =	6 ×7
7 ×6	8×6 =	6×8 =	7 ×6
8×6 =	6 ×7	8 ×6	8×6 =
6 ×8	8×6 =	6×7 =	7×6 =
6 ×7	6×7 =	6 ×8	8 ×6
8×7 =	3 ×4	7 ×8	4×3 =
4×3 =	3×4 =	8×7 =	8 ×7
8 ×7	7×8 =	4 ×3	4×3 =
3 ×4	3×4 =	7×8 =	3 ×4
7×8 =	7 ×8	4 ×3	8 ×7

Multiplication Fact Linking #23 Name _____

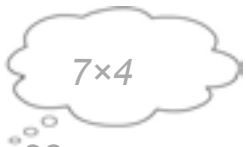
Mixed Facts Practice Page: The Last Ten Multiplication Facts – All parts Answer as many as possible. Look for swaps (for example, 3×8 is the same as 8×3).

.

3×8 =	8 ×3	3×4 =	4 ×3
6 ×3	3×6 =	8×7 =	7 ×8
6 ×4	7 ×3	3 ×7	4×7 =
7×8 =	6×7 =	8 ×6	4×8 =
3 ×8	6×8 =	3 ×6	7 ×6
6×3 =	4 ×7	8 ×4	7×4 =
7×6 =	4×6 =	3×7 =	6 ×8
8 ×7	4×3 =	4 ×6	8×4 =
8×6 =	6×4 =	7 ×4	6 ×7
7×3 =	4 ×8	8×3 =	3 ×4

Multiplication Fact Linking #24 Name _____

Think Page: Last Ten Multiplication Facts In the "think cloud" write the swap for each problem (example: the swap for 4×7 is 7×4). Then answer the problem.



4×7 = 28

7×4 =

6×7 =

4×3 =

3×6 =

4×6 =

7×3 =

8×3 =

3×4 =

7×8 =

6×8 =

8×7 =

7×6 =

3×6 =

6×4 =

8×4 =

6×3 =

3×7 =

3×8 =

8×7 =

8×6 =

7×6 =

4×8 =

6×4 =

6×8 =

8×3 =

4×3 =

7×4 =

Multiplication Fact Linking #25 Name _____

Think Page: Last Ten Multiplication Facts In the "think cloud" write the swap for each problem (example: the swap for 4×7 is 7×4). Then answer the problem.

7 ×8 56  8 ×7	3 ×4 	8 ×6 	4 ×8 	3 ×7 
7 ×6 	6 ×4 	8 ×3 	6 ×7 	3 ×6 
6 ×8 	7 ×3 	8 ×7 	4 ×7 	8 ×4 
8 ×7 	6 ×3 	4 ×6 	4 ×8 	7 ×4 
6 ×4 	7 ×6 	4 ×3 	3 ×8 	6 ×3 

Multiplication Fact Linking #26 Name _____

One-Minute Progress Check: The Last Ten Multiplication Facts Complete as many of these problems as possible in 1 minute.

4×3 =	3 ×4	4 ×7	7×6 =
8×7 =	6×8 =	4×6 =	7 ×8
7×3 =	4 ×8	6 ×8	8 ×7
4×7 =	7 ×6	3×4 =	3×6 =
6 ×4	3×8 =	7 ×3	8 ×4
3×7 =	8×6 =	6×4 =	6 ×7
8×4 =	7 ×4	4 ×6	6 ×3
6×7 =	8 ×3	3 ×8	3 ×7
3 ×6	6×3 =	8 ×6	7×4 =
7×8 =	4 ×3	4×8 =	8×3 =

Division Fact Linking #1 Name _____

Think Page: ÷2 In the "think cloud" write the linking ×2 fact for each problem (example: 18÷9=❑ links to 9×❑=18, so the linking ×2 fact for 18÷9 is 9×2). Then answer the problem.

18÷2 = 9

6÷2 =

14÷7 =

8÷4 =

20÷2 =

12÷2 =

10÷2 =

14÷2 =

16÷8 =

8÷2 =

12÷6 =

20÷10 =

14÷7 =

10÷5 =

16÷2 =

18÷9 =

16÷8 =

12÷2 =

4÷2 =

6÷3 =

14÷2 =

18÷2 =

20÷2 =

8÷4 =

16÷8 =

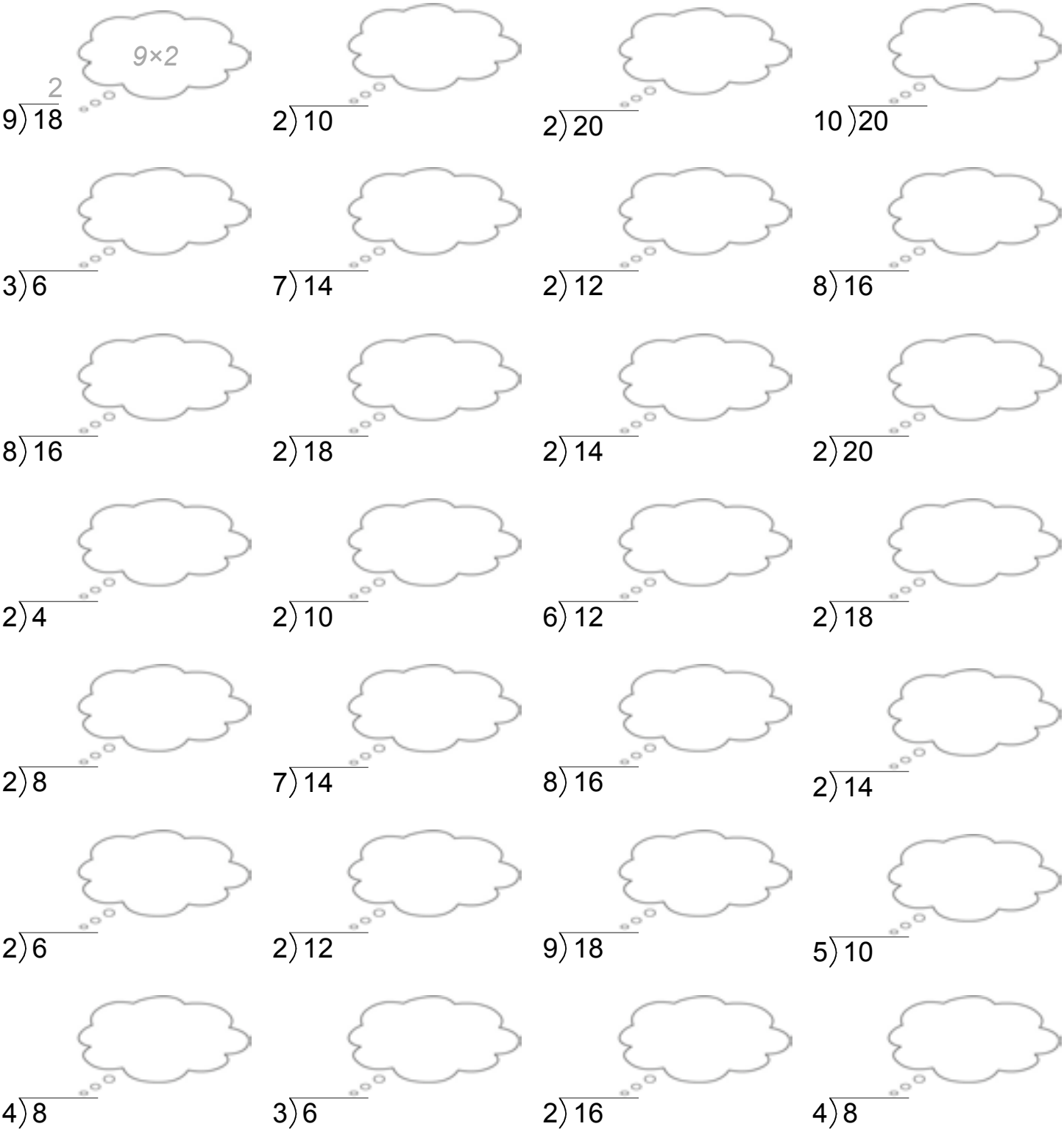
6÷3 =

18÷9 =

10÷2 =

Division Fact Linking #2 Name _____

Think Page: ÷2 In the "think cloud" write the linking ×2 fact for each problem (example: $9\overline{)18}$ links to 9×❑=18, so the linking ×2 fact for $9\overline{)18}$ is 9×2). Then answer the problem.



Division Fact Linking #3 Name _____

Practice Page: ÷2 Link to base facts: ×2. (Remember, 6÷3=❑ is the same as 3×❑=6).

4÷2 =	$10\overline{)20}$	6÷3 =	$8\overline{)16}$
12÷2 =	$2\overline{)10}$	18÷9 =	$4\overline{)8}$
$7\overline{)14}$	10÷5 =	16÷8 =	20÷10 =
$2\overline{)18}$	$6\overline{)12}$	$2\overline{)6}$	10÷2 =
14÷7 =	$2\overline{)16}$	6÷3 =	8÷2 =
$2\overline{)14}$	6÷2 =	$5\overline{)10}$	$4\overline{)8}$
$3\overline{)6}$	14÷2 =	$2\overline{)12}$	$2\overline{)4}$
14÷7 =	10÷5 =	18÷2 =	16÷2 =
$2\overline{)16}$	12÷6 =	20÷2 =	$2\overline{)8}$
8÷4 =	$2\overline{)20}$	$6\overline{)12}$	$9\overline{)18}$

Division Fact Linking #4 Name _____

Mixed Fact Group Practice Page: ×2 and ÷2 Answer as many as possible. For division, think the reverse of multiplication (for example, for 6÷3=❑ think 3×❑=6).

3×2 =	6÷3 =	2 ×3	$2\overline{)8}$
6 ×2	5 ×2	12÷2 =	2×4 =
10 ×2	10÷5 =	2 ×8	$8\overline{)16}$
14÷2 =	2×2 =	$2\overline{)10}$	2 ×5
2×9 =	9 ×2	$2\overline{)18}$	$2\overline{)4}$
7 ×2	4 ×2	2 ×2	16÷2 =
$7\overline{)14}$	8÷4 =	2×6 =	$2\overline{)6}$
2×9 =	12÷2 =	20÷10 =	8×2 =
2×10 =	$6\overline{)12}$	5×2 =	4÷2 =
2×7 =	14÷2 =	$2\overline{)20}$	18÷9 =

Division Fact Linking #5 Name _____

One-Minute Progress Check: ÷2 Complete as many of these problems as possible in 1 minute.

12÷2 =	$5\overline{)10}$	20÷2 =	$8\overline{)16}$
4÷2 =	16÷2 =	8÷4 =	6÷2 =
$4\overline{)8}$	$2\overline{)12}$	$3\overline{)6}$	20÷2 =
10÷5 =	18÷2 =	$7\overline{)14}$	10÷2 =
14÷2 =	16÷8 =	6÷3 =	$2\overline{)8}$
$9\overline{)18}$	6÷3 =	$2\overline{)4}$	$10\overline{)20}$
6÷2 =	12÷2 =	$2\overline{)14}$	12÷6 =
$2\overline{)10}$	$2\overline{)20}$	18÷9 =	$2\overline{)16}$
20÷10 =	$6\overline{)12}$	$2\overline{)6}$	8÷2 =
$4\overline{)8}$	$2\overline{)14}$	14÷7 =	$2\overline{)18}$

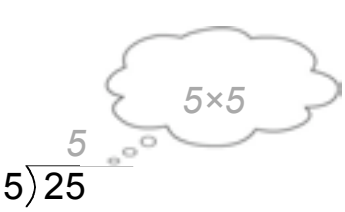
Division Fact Linking #6 Name _____

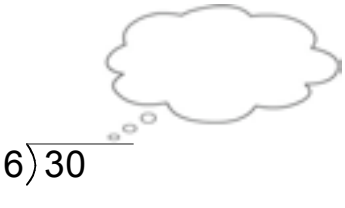
Think Page: ÷5 Identify the linking ×5 fact for each problem (example: 15÷5=❑ links to 5×❑=15, so the linking ×5 fact for 15÷5 is 5×3).

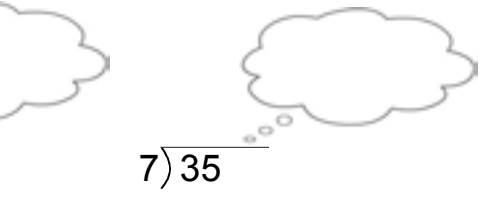
15÷5 = *3* (*5×3*)	40÷8 =	15÷3 =	20÷4 =
20÷4 =	45÷5 =	10÷2 =	35÷5 =
30÷5 =	25÷5 =	50÷10 =	40÷5 =
35÷7 =	40÷8 =	15÷5 =	30÷6 =
40÷5 =	30÷5 =	45÷9 =	35÷7 =
35÷5 =	20÷5 =	50÷5 =	30÷6 =
45÷5 =	10÷5 =	15÷3 =	25÷5 =

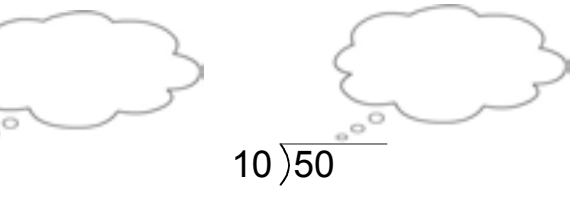
Division Fact Linking #7 Name _____

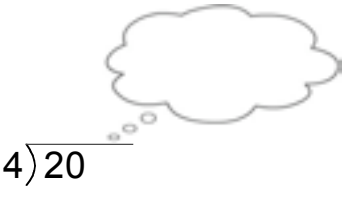
Think Page: ÷5 In the "think cloud" write the linking ×5 fact for each problem (example: $5\overline{)20}$ links to 5×❑=20, so the linking ×5 fact for $5\overline{)20}$ is 5×4). Then answer the problem.

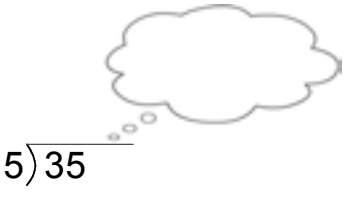


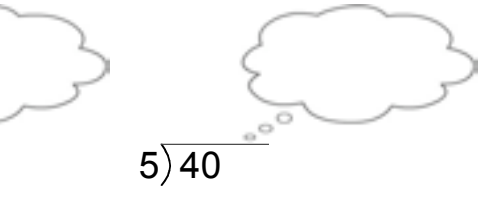


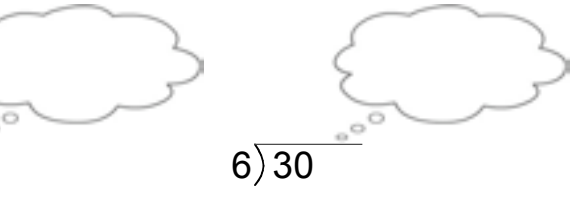


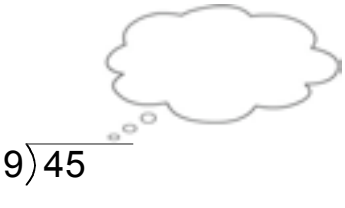


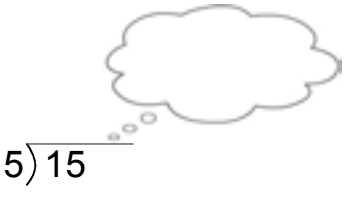


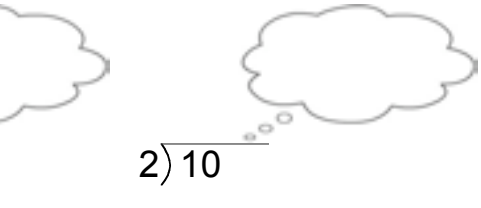


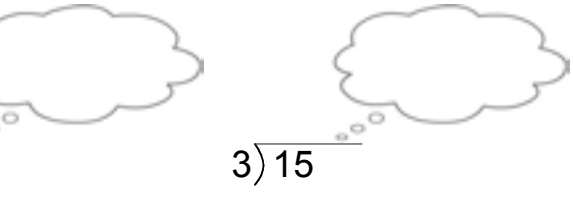


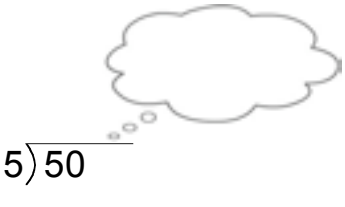


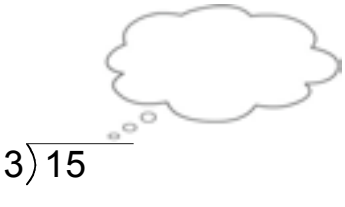


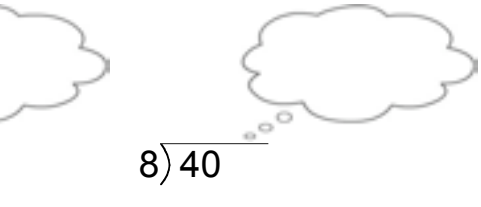


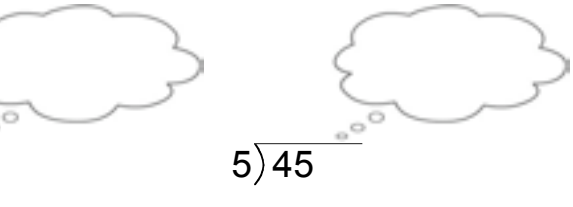


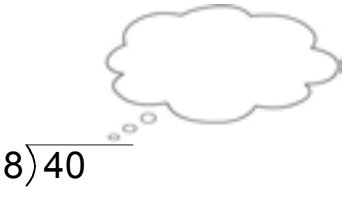


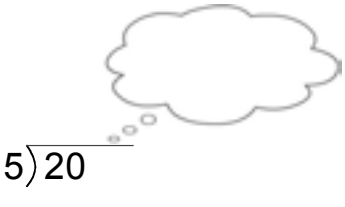


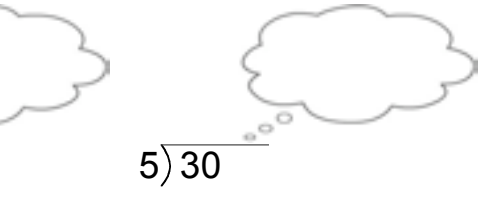


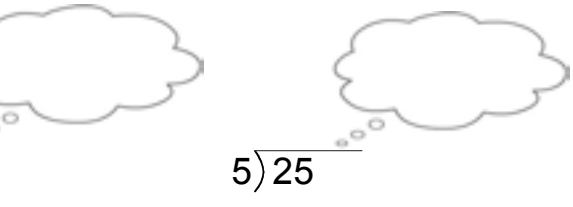


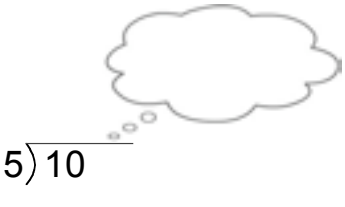


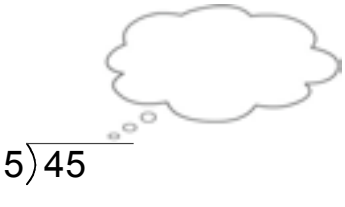


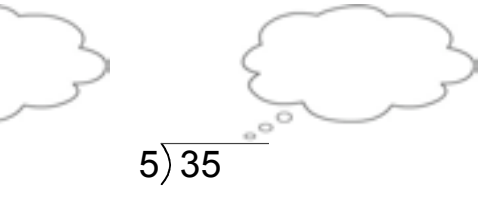


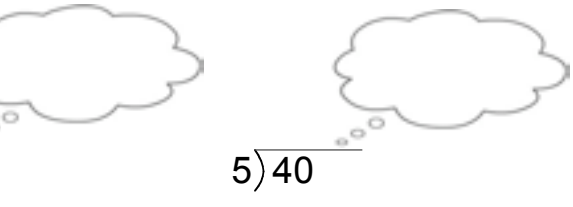


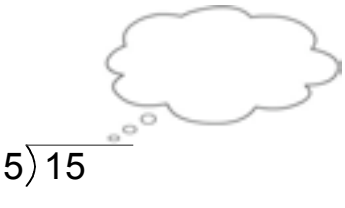


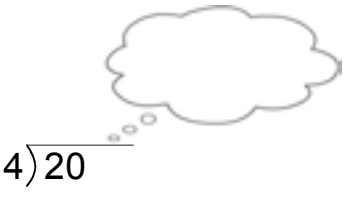


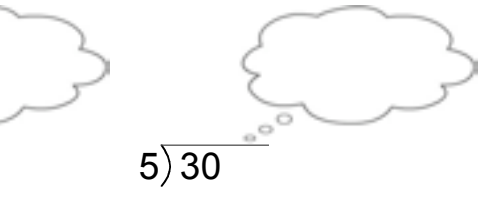


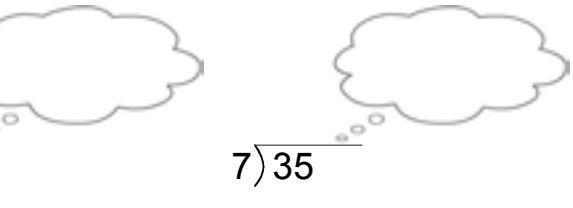












Division Fact Linking #8 Name _____

Mixed Fact Group Practice Page: ×5 and ÷5 Answer as many as possible. For division, think the reverse of multiplication (for example, for 20÷5=❑ think 5×❑=20).

$5\overline{)20}$	$\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$	40÷8 =	5×8 =
5×2 =	$9\overline{)45}$	25÷5 =	$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$
30÷6 =	$5\overline{)30}$	5×6 =	5×10 =
$\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$	10÷5 =	15÷3 =	$\begin{array}{r} 10 \\ \times 5 \\ \hline \end{array}$
35÷7 =	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$	$5\overline{)50}$	45÷5 =
5×7 =	$5\overline{)25}$	5×5 =	$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$
$5\overline{)40}$	9×5 =	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$
5×3 =	50÷10 =	$2\overline{)10}$	$8\overline{)40}$
$5\overline{)15}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$5\overline{)35}$	45×5 =
30÷6 =	20÷4 =	15÷3 =	$5\overline{)35}$

Division Fact Linking #9 Name _____

Practice Page: ÷5 Link to base facts: ×5. (Remember, 30÷5=❑ links to 5×❑=30).

30÷5 =	$7\overline{)35}$	40÷8 =	45÷9 =
45÷9 =	50÷5 =	$5\overline{)30}$	$5\overline{)25}$
20÷5 =	35÷5 =	15÷3 =	$10\overline{)50}$
$9\overline{)45}$	15÷5 =	$4\overline{)20}$	50÷5 =
$2\overline{)10}$	20÷4 =	$8\overline{)40}$	30÷6 =
$6\overline{)30}$	$5\overline{)35}$	50÷10 =	$3\overline{)15}$
$5\overline{)15}$	10÷2 =	25÷5 =	$5\overline{)10}$
35÷7 =	$5\overline{)45}$	$5\overline{)50}$	40÷5 =
10÷5 =	$3\overline{)15}$	30÷ 5 =	$5\overline{)20}$
$5\overline{)40}$	20÷4 =	$5\overline{)35}$	$5\overline{)40}$

Division Fact Linking #10 Name _____

One-Minute Progress Check: ÷5 Complete as many of these problems as possible in 1 minute.

$5\overline{)35}$	30÷5 =	$5\overline{)50}$	25÷5 =
15÷3 =	$4\overline{)20}$	$8\overline{)40}$	50÷10 =
$5\overline{)30}$	40÷8 =	$7\overline{)35}$	50÷5 =
$5\overline{)35}$	20÷4 =	15÷5 =	35÷5 =
10÷2 =	$5\overline{)45}$	$3\overline{)15}$	20÷4 =
$5\overline{)15}$	35÷7 =	10÷5 =	$5\overline{)40}$
$6\overline{)30}$	$2\overline{)10}$	$9\overline{)45}$	20÷5 =
$5\overline{)25}$	45÷9 =	30÷5 =	45÷9 =
$10\overline{)50}$	50÷5 =	30÷6 =	$3\overline{)15}$
$5\overline{)40}$	$5\overline{)20}$	40÷5 =	$5\overline{)10}$

Division Fact Linking #11 Name _____

Think Page: ÷9 In the "think cloud" write the linking ×9 fact for each problem (example: 27÷9=❑ links to 9×❑=27, so the linking ×9 fact for 27÷9 is 9×3). Then answer the problem.

27÷9 = 3

63÷9 =

18÷9 =

27÷3 =

81÷9 =

54÷6 =

45÷5 =

72÷9 =

36÷4 =

72÷8 =

54÷9 =

63÷7 =

45÷9 =

90÷9 =

72÷9 =

45÷5 =

45÷5 =

54÷6 =

36÷9 =

63÷7 =

36÷9 =

90÷10 =

63÷9 =

36÷4 =

27÷9 =

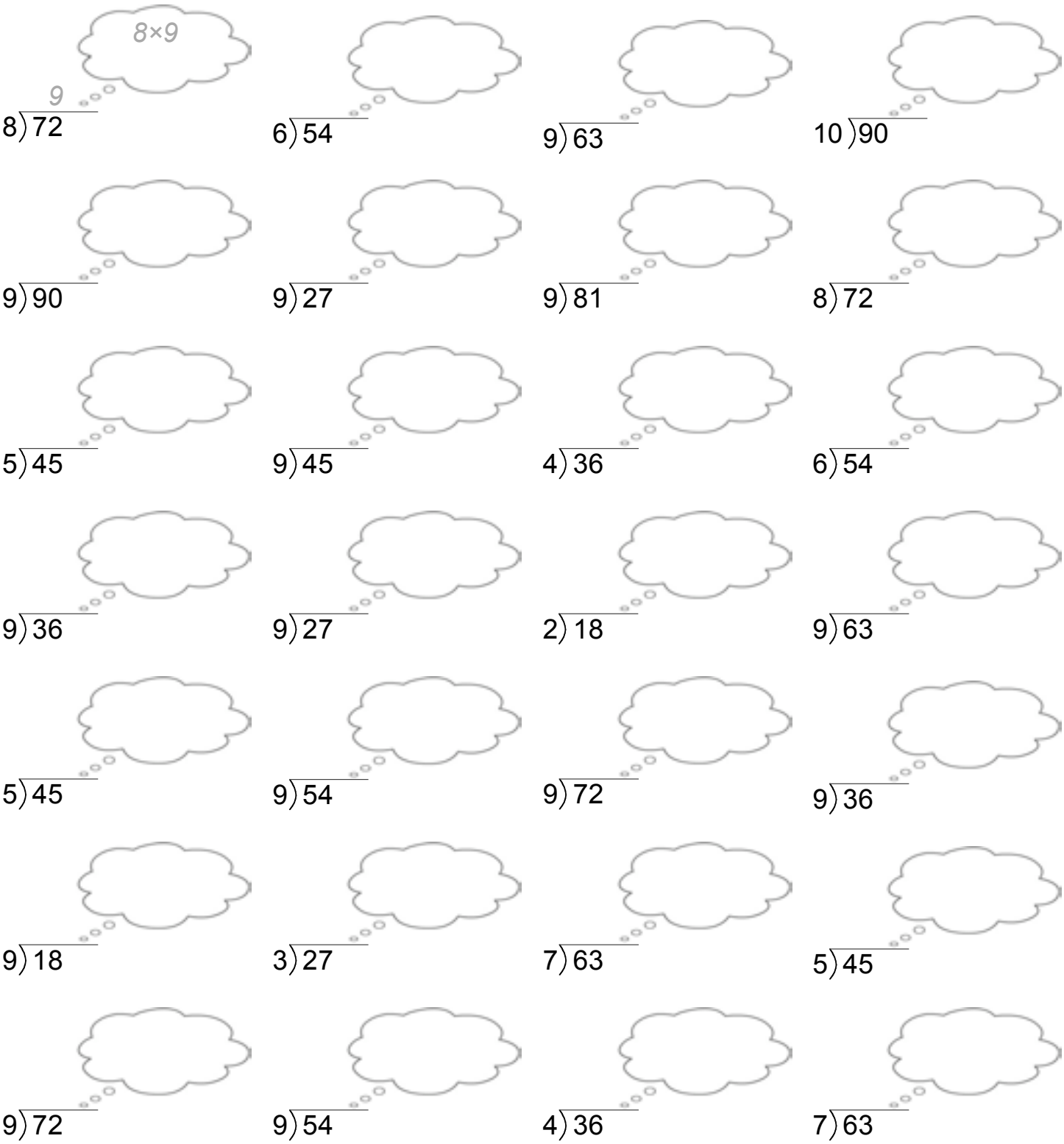
72÷8 =

18÷2 =

54÷9 =

Division Fact Linking #12 Name _____

Think Page: ÷9 In the "think cloud" write the linking ×9 fact for each problem (example: $9\overline{)36}$ links to 9×❑=36, so the linking ×9 fact for $9\overline{)36}$ is 9×4). Then answer the problem.



Division Fact Linking #13 Name _____

Mixed Fact Group Practice Page: ×9 and ÷9 Answer as many as possible. For division, think the reverse of multiplication (for example, for 27÷9=❑ think 9×❑=27).

27÷9 =	9 ×3	45÷9 =	9×5 =
2 ×9	9⟌36	81÷9 =	9×9 =
6⟌54	54÷9 =	9 ×6	9×10 =
9×2 =	18÷9 =	3⟌27	10 ×9
63÷7 =	7 ×9	9⟌63	5⟌45
9×7 =	81÷9 =	5 ×9	8 ×9
72÷9 =	9×8 =	9 ×9	9×4 =
3×9 =	90÷10 =	2⟌18	9⟌81
27÷9 =	9⟌54	8⟌72	10⟌90
4 ×9	36÷4 =	6×9 =	63÷7 =

Division Fact Linking #14 Name _____

Practice Page: ÷9 Link to base facts: ×9. (Remember, 36÷9=❑ links to 9×❑=36).

54÷6 =	$9\overline{)72}$	$7\overline{)63}$	63÷9 =
$9\overline{)45}$	45÷5 =	54÷9 =	$4\overline{)36}$
$9\overline{)18}$	$9\overline{)81}$	$6\overline{)54}$	36÷9 =
36÷9 =	36÷4 =	63÷7 =	$9\overline{)72}$
$9\overline{)63}$	$9\overline{)54}$	45÷9 =	$9\overline{)27}$
$8\overline{)72}$	36÷4 =	27÷9 =	90÷10 =
$3\overline{)27}$	18÷2 =	90÷9 =	45÷5 =
72÷9 =	$9\overline{)63}$	$9\overline{)36}$	72÷8 =
18÷9 =	$5\overline{)45}$	54÷6 =	$10\overline{)90}$
81÷9 =	27÷3 =	$2\overline{)18}$	36÷9 =

Division Fact Linking #15 Name _____

One-Minute Progress Check: ÷9 Complete as many of these problems as possible in 1 minute.

27÷9 =	18÷2 =	$9\overline{)63}$	$9\overline{)36}$
63÷7 =	$9\overline{)81}$	72÷8 =	$6\overline{)54}$
$9\overline{)90}$	$3\overline{)27}$	36÷4 =	$5\overline{)45}$
$9\overline{)18}$	45÷9 =	$7\overline{)63}$	72÷9 =
36÷9 =	$9\overline{)27}$	54÷6 =	90÷9 =
81÷9 =	$9\overline{)72}$	$2\overline{)18}$	$8\overline{)72}$
$9\overline{)54}$	27÷3 =	$7\overline{)63}$	81÷9 =
90÷10 =	$9\overline{)45}$	18÷9 =	90÷9 =
36÷4 =	45÷5 =	54÷9 =	$4\overline{)36}$
$10\overline{)90}$	$9\overline{)72}$	63÷9 =	36÷9 =

Division Fact Linking #16 Name _____

Base Facts Practice Page: ÷1 (1-100) Answer as many as possible. Dividing by 1 (÷1) is the same as multiplying by 1 (×1).

29÷1 =	62÷1 =	66÷1 =	79÷1 =
22÷1 =	77÷1 =	49÷1 =	44÷1 =
69÷1 =	89÷1 =	30÷1 =	20÷1 =
26÷1 =	4÷1 =	16÷1 =	14÷1 =
99÷1 =	2÷1 =	55÷1 =	7÷1 =
39÷1 =	47÷1 =	63÷1 =	28÷1 =
35÷1 =	97÷1 =	14÷1 =	22÷1 =
59÷1 =	53÷1 =	15÷1 =	31÷1 =
71÷1 =	19÷1 =	62÷1 =	56÷1 =
48÷1 =	10÷1 =	80÷1 =	33÷1 =

Division Fact Linking #17 Name _____

Base Facts Practice Page: ÷1 (1-1000) Answer as many as possible. Dividing by 1 (÷1) is the same as multiplying by 1 (×1).

199÷1 =	765÷1 =	121÷1 =	69÷1 =
876÷1 =	531÷1 =	881÷1 =	210÷1 =
794÷1 =	499÷1 =	923÷1 =	82÷1 =
892÷1 =	909÷1 =	19÷1 =	444÷1 =
321÷1 =	987÷1 =	543÷1 =	328÷1 =
299÷1 =	699÷1 =	11÷1 =	432÷1 =
115÷1 =	416÷1 =	402÷1 =	654÷1 =
201÷1 =	381÷1 =	628÷1 =	164÷1 =
599÷1 =	326÷1 =	706÷1 =	177÷1 =
432÷1 =	234÷1 =	548÷1 =	830÷1 =

Division Fact Linking #18 Name _____

One-Minute Progress Check: ÷1 Complete as many of these problems as possible in 1 minute.

199÷1 =	765÷1 =	708÷1 =	19÷1 =
876÷1 =	543÷1 =	54÷1 =	210÷1 =
799÷1 =	183÷1 =	77÷1 =	99÷1 =
890÷1 =	353÷1 =	231÷1 =	606÷1 =
321÷1 =	604÷1 =	555÷1 =	316÷1 =
299÷1 =	21÷1 =	475÷1 =	539÷1 =
119÷1 =	490÷1 =	861÷1 =	66÷1 =
210÷1 =	727÷1 =	302÷1 =	903÷1 =
599÷1 =	554÷1 =	244÷1 =	13÷1 =
432÷1 =	88÷1 =	668÷1 =	89÷1 =

Division Fact Linking #19 Name _____

Think Page: Division Squares In the "think cloud" write the linking multiplication square for each problem (example: 36÷6=❑ links to 6×❑=36, so the linking multiplication square for 36÷6 is 6×6). Then answer the problem.

36÷6 = *6*

16÷4 =

49÷7 =

9÷3 =

81÷9 =

64÷8 =

25÷5 =

81÷9 =

16÷4 =

36÷6 =

49÷7 =

100÷10 =

49÷7 =

9÷3 =

64÷8 =

81÷9 =

25÷5 =

36÷6 =

49÷7 =

9÷3 =

49÷7 =

100÷10 =

16÷4 =

36÷6 =

9÷3 =

64÷8 =

25÷5 =

81÷9 =

Division Fact Linking #20 Name _____

Think Page: Division Squares In the "think cloud" write the linking multiplication square for each problem (example: $6\overline{)36}$ links to 6×❑=36, so the linking multiplication square for $6\overline{)36}$ is 6×6). Then answer the problem.

$5\overline{)25}$ $7\overline{)49}$ $3\overline{)9}$ $10\overline{)100}$

$6\overline{)36}$ $4\overline{)16}$ $9\overline{)81}$ $6\overline{)36}$

$9\overline{)81}$ $8\overline{)64}$ $7\overline{)49}$ $5\overline{)25}$

$6\overline{)36}$ $3\overline{)9}$ $9\overline{)81}$ $4\overline{)16}$

$4\overline{)16}$ $6\overline{)36}$ $8\overline{)64}$ $6\overline{)36}$

$7\overline{)49}$ $9\overline{)81}$ $4\overline{)16}$ $9\overline{)81}$

$8\overline{)64}$ $5\overline{)25}$ $6\overline{)36}$ $3\overline{)9}$

Division Fact Linking #21 Name _____

Mixed Fact Group Practice Page: Multiplication and Division Squares Answer as many as possible. For division, think the reverse of multiplication (for example, for 36÷6=❑ think 6×❑=36).

81÷9 =	3×3 =	64÷8 =	6×6 =
7 ×7	6 ×6	25÷5 =	5×5 =
36÷6 =	$5\overline{)25}$	4×4 =	$7\overline{)49}$
3 ×3	$9\overline{)81}$	9÷3 =	5 ×5
49÷7 =	8 ×8	$8\overline{)64}$	$2\overline{)4}$
$4\overline{)16}$	25÷5 =	1×1 =	9 ×9
25÷5 =	$3\overline{)9}$	4 ×4	2×2 =
7×7 =	$6\overline{)36}$	4÷2 =	16÷4 =
36÷6 =	2 ×2	$7\overline{)49}$	9×9 =
8 ×8	16÷4 =	8×8 =	$8\overline{)64}$

Division Fact Linking #22 Name _____

Practice Page: Division Squares Link to base facts: multiplication squares (remember, 9÷3=❑ links to 3×❑=9).

36÷6 =	$5\overline{)25}$	49÷7 =	$6\overline{)36}$
49÷7 =	64÷8 =	$4\overline{)16}$	64÷8 =
16÷4 =	$6\overline{)36}$	$3\overline{)9}$	81÷9 =
25÷5 =	$9\overline{)81}$	$7\overline{)49}$	$5\overline{)25}$
$7\overline{)49}$	$8\overline{)64}$	9÷3 =	$4\overline{)16}$
81÷9 =	$3\overline{)9}$	25÷5 =	49÷7 =
9÷3 =	36÷6 =	9÷3 =	$6\overline{)36}$
$5\overline{)25}$	$8\overline{)64}$	81÷9 =	64÷8 =
$7\overline{)49}$	36÷6 =	$3\overline{)9}$	16÷4 =
$9\overline{)81}$	$8\overline{)64}$	49÷7 =	$9\overline{)81}$

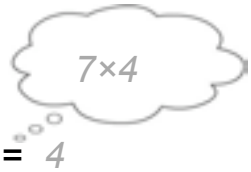
Division Fact Linking #23 Name _____

One-Minute Progress Check: Division Squares Complete as many of these problems as possible in 1 minute.

$8\overline{)64}$	36÷6 =	81÷9 =	$8\overline{)64}$
25÷5 =	9÷3 =	$4\overline{)16}$	25÷5 =
16÷4 =	$9\overline{)81}$	64÷8 =	49÷7 =
100÷10 =	16÷4 =	$6\overline{)36}$	$5\overline{)25}$
49÷7 =	$8\overline{)64}$	9÷3 =	16÷4 =
$6\overline{)36}$	$3\overline{)9}$	25÷5 =	9÷3 =
9÷3 =	$7\overline{)49}$	36÷6 =	49÷7 =
49÷7 =	$5\overline{)25}$	81÷9 =	64÷8 =
100÷10 =	$6\overline{)36}$	$4\overline{)16}$	$3\overline{)9}$
$8\overline{)64}$	$10\overline{)100}$	$7\overline{)49}$	$9\overline{)81}$

Division Fact Linking #24 Name _____

Think Page: Last Ten Division Facts In the "think cloud" write the linking last ten *multiplication* fact for each problem (example: 28÷7=❑ links to 7×❑=28, so the linking multiplication fact for 28÷7 is 7×4). Then answer the problem.



28÷7 = 4

28÷4 =

42÷7 =

12÷3 =

18÷6 =

24÷6 =

21÷3 =

24÷3 =

12÷4 =

56÷8 =

48÷8 =

56÷7 =

42÷6 =

18÷6 =

24÷4 =

32÷4 =

18÷3 =

21÷7 =

24÷8 =

56÷7 =

48÷6 =

42÷6 =

32÷8 =

24÷4 =

48÷8 =

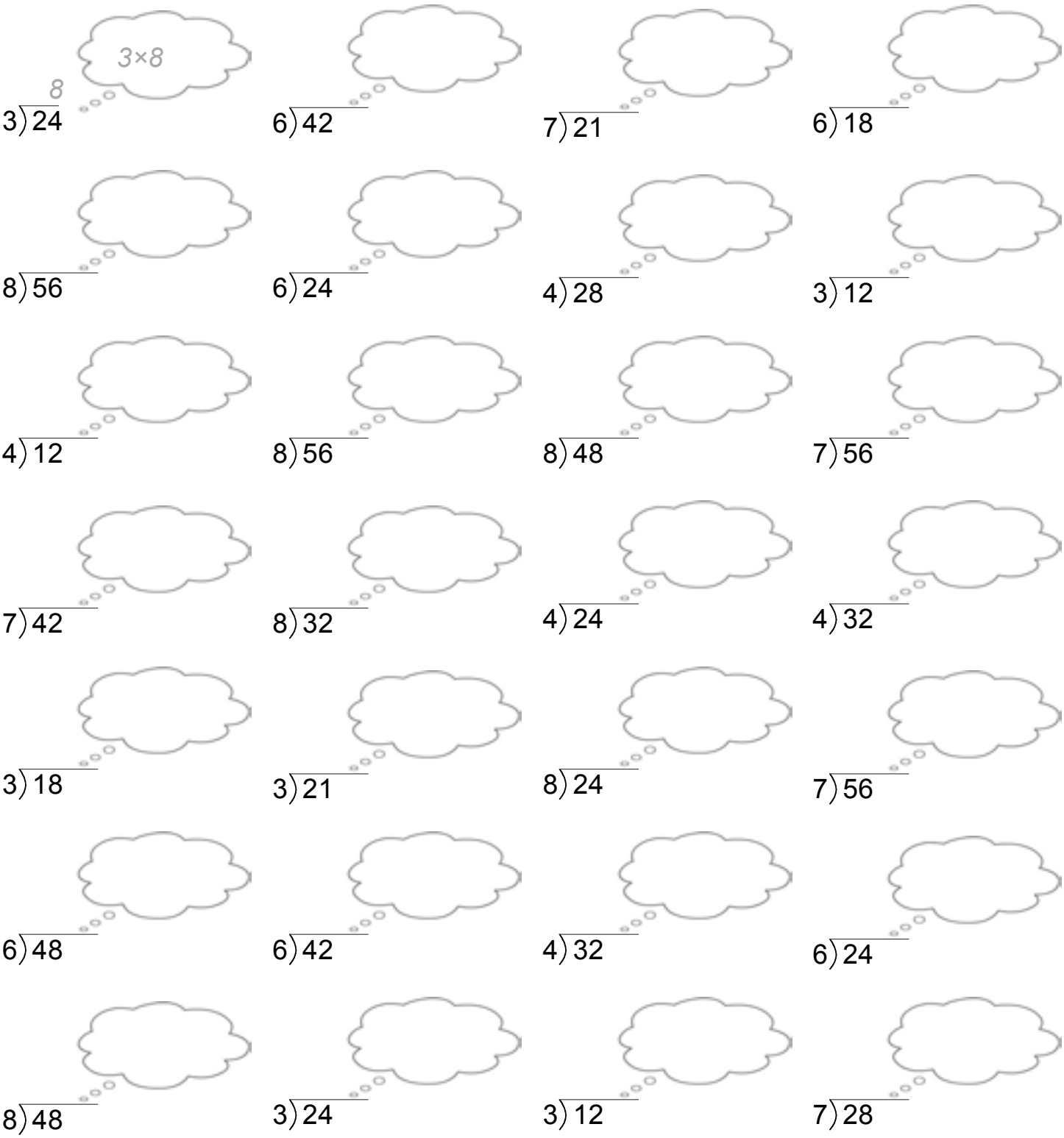
24÷3 =

12÷3 =

28÷4 =

Division Fact Linking #25 Name _____

Think Page: Last Ten Division Facts In the "think cloud" write the linking last ten *multiplication* fact for each problem (example: $3\overline{)24}$ links to 3×❑=24, so the linking multiplication square for $3\overline{)24}$ is 3×8). Then answer the problem.



Division Fact Linking #26 Name _____

Mixed Fact Group Practice Page: Last Ten Multiplication and Division Facts

Answer as many as possible. For division, think the reverse of multiplication (for example, for 18÷3=❑ think 3×❑=18).

18÷3 =	3×6 =	$7\overline{)42}$	6×7 =
48÷8 =	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$	$7\overline{)28}$	4×7 =
$6\overline{)48}$	8×4 =	7×3 =	$7\overline{)21}$
$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$	32÷8 =	24÷3 =	$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$
56÷7 =	$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$	$6\overline{)24}$	$3\overline{)12}$
$\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$	28÷4 =	6×4 =	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$
$8\overline{)56}$	8×7 =	$\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$	8×6 =
3×4 =	24÷4 =	3×8 =	21÷3 =
$8\overline{)24}$	$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	42÷6 =	$\begin{array}{r} 4 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ \times 7 \\ \hline \end{array}$	$4\overline{)32}$	$6\overline{)18}$	12÷4 =

Division Fact Linking #27 Name _____

Practice Page: The Last Ten Division Facts Link to base facts: last ten multiplication facts (remember, 24÷8=❑ links to 8×❑=24).

24÷8 =	24÷3 =	$4\overline{)12}$	$3\overline{)12}$
18÷3 =	18÷6 =	$6\overline{)24}$	56÷8 =
$3\overline{)24}$	$3\overline{)21}$	$7\overline{)21}$	28÷7 =
$8\overline{)56}$	42÷7 =	$8\overline{)48}$	32÷8 =
$8\overline{)24}$	48÷8 =	$6\overline{)18}$	$6\overline{)42}$
$3\overline{)18}$	$7\overline{)28}$	24÷4 =	28÷4 =
42÷6 =	24÷6 =	21÷3 =	56÷7 =
$7\overline{)56}$	12÷3 =	$4\overline{)24}$	$4\overline{)32}$
48÷6 =	32÷4 =	$4\overline{)28}$	$7\overline{)42}$
21÷7 =	$8\overline{)32}$	$6\overline{)48}$	12÷4 =

Division Fact Linking #28 Name _____

One-Minute Progress Check: The Last Ten Division Facts Complete as many of these problems as possible in 1 minute.

12÷3 =	$4\overline{)12}$	28÷4 =	$6\overline{)42}$
56÷7 =	48÷8 =	24÷6 =	$7\overline{)56}$
$3\overline{)21}$	32÷8 =	$8\overline{)48}$	56÷8 =
$7\overline{)28}$	42÷6 =	12÷4 =	18÷6 =
24÷4 =	$8\overline{)24}$	21÷3 =	$4\overline{)32}$
21÷7 =	$6\overline{)48}$	$8\overline{)32}$	42÷7 =
$6\overline{)24}$	$4\overline{)28}$	$4\overline{)24}$	$3\overline{)18}$
$7\overline{)42}$	24÷3 =	24÷8 =	$7\overline{)21}$
$6\overline{)18}$	18÷3 =	48÷6 =	28÷7 =
$8\overline{)56}$	$3\overline{)12}$	32÷4 =	$3\overline{)24}$